Math 256. Midterm exam.

No formula sheet, books or calculators! Include this answer sheet with your answer booklet to receive credit for part I!

Name:

Part I

Circle what you think is the correct answer. +3 for a correct answer, −1 for a wrong answer, 0 for no answer.

1. The integrating factor for the ODE $y' + y\cot x = e^x(\sin x)^{-1}$ is

(a) $\sin x$ (b) $\cos x$ (c) $\exp(\cos x)$ (d) $\ln(\cos x)$ (e) *None of the above.*

2. The solution to the ODE $y' + y\cot x = 2x(\sin x)^{-1}$ is

(a) $\dfrac{C+x}{\cos x}$ (b) $\dfrac{C+x}{\sin x}$ (c) $\dfrac{x^2-C}{\sin x}$ (d) $\dfrac{x^2+C}{\cos x}$ (e) *None of the above,*

where C is an arbitrary constant.

3. The ODE $y'\sin y + \cos x = 0$, with $y(0) = 0$, has the solution,

(a) $\cos^{-1}(\sin x + 1)$ (b) $\cos(\pi \sin x)$ (c) $\cos^{-1}(\sin x)$

(d) $x\cos x$ (e) *None of the above.*

4. The ODE $y'' + 4y' + 5y = 85\sin x$, has the homogeneous solution,

(a) $e^{-2x}(A\cos 2x + B\sin 2x)$ (b) $e^{-2x}(A\cos x + B\sin x)$ (c) $Ae^{-2x}\cos(3x + B)$

(d) $Ae^{(1+3i)x} + Be^{(-1+3i)x}$ (e) *None of the above,*

where A and B are arbitrary constants.

5. The particular solution to $y'' + 4y' + 5y = 85\sin x$, is

(a) $9\sin x$ (b) $9\cos x$ (c) $2\sin x$

(d) $2\cos x$ (e) *None of the above.*

Part II

Answer in full (i.e. give as many arguments, explanations and steps as you think is needed for a normal person to understand your logic). Answer as much as you can; partial credit awarded.

1. *(8 marks)* (a) By treating the problem as separable, solve the first-order ODE

$$y' = (3 - \sin x)(3 - y).$$

(b) Now solve the problem again using an integrating factor.

2. *(8 marks)* Solve the ODE,

$$y'' - 4y' + 5y = xe^{2x}.$$

Without solving the problem, indicate what trial particular solution you would have chosen had the right-hand side been one of the homogeneous solutions.

Helpful trig identities:

$$\sin 0 = \sin \pi = 0, \quad \sin(\pi/2) = 1 = -\sin(3\pi/2),$$

$$\cos 0 = -\cos \pi = 1, \quad \cos(\pi/2) = \cos(3\pi/2) = 0,$$

$$\sin(-A) = -\sin A, \quad \cos(-A) = \cos A, \quad \sin^2 A + \cos^2 A = 1,$$

$$\sin(2A) = 2\sin A \cos A, \quad \sin(A + B) = \sin A \cos B + \cos A \sin B,$$

$$\cos(2A) = \cos^2 A - \sin^2 A, \quad \cos(A + B) = \cos A \cos B - \sin A \sin B,$$

$$\frac{d}{dx}\sin x = \cos x, \quad \frac{d}{dx}\cos x = -\sin x.$$