I.1 $\int \frac{\cos x\,dx}{\sin x} = \ln(\sin x)$ A23i

$\Rightarrow I = \sin x$ (a)

2. $(\sin x\, y)' = 2x \quad \therefore \quad \frac{x^2 + \text{con}}{\sin x} = y$ (c)

3. $-\cos y + \sin x = \text{Con.} = -1$ if $y(0) = 0$

$\therefore y = \cos^{-1}(1 + \sin x)$ (a)

4. Aux Eq $m^2 + 4m + 5 = 0$, $(m+2)^2 = -1$, $m = -2 \pm i$ (b)

5. Need $d_1 \cos x + d_2 \sin x$ $\quad 4d_1 \cos x + 4d_2 \sin x + 4(-d_1 \sin x + d_2 \cos x) = 8S \sin x$

$d_1 = d_2 \Rightarrow$ (e)

II. 1a $\frac{y'}{3-y} = 3 - \sin x \Rightarrow -\ln|3-y| = 3x + \cos x + \text{con.}$ 2 for two integrals

1 for rewrite

$\therefore y = 3 + Ae^{-3x - \cos x}$ 1 for final ans.

1b. $y' + (3 - \sin x)y = 3(3 - \sin x)$ $\Big\} \rightarrow y = \frac{C}{I} + \frac{3}{I}\int pI\,dx$

$I = \exp(3x + \cos x)$

2 — 1 for rewrite, 1 for I

$3 + Ce^{-3x - \cos x}$

2 — 1 for 2nd int., 1 for ans.

2. Homog. Sols: Aux Eq $m^2 - 4m + 5 = 0$

$(m-2)^2 = -1$ $\quad m = 2 \pm i$

$\therefore y_h = (C\cos x + D\sin x)e^{2x}$ 3 for hom sols

Trial Part. Sol.: $y_p = (d_1 x + d_2)e^{2x}$

$y_p' = (d_1 + 2d_1 x + 2d_2)e^{2x}$, $y_p'' = (4d_1 + 4d_1 x + 4d_2)e^{2x}$

$\rightarrow (\cancel{4d_1} + 4d_1 x + 4d_2 - \cancel{4d_1} - 8d_1 x - 8d_2 + 5d_1 x + 5d_2)\cancel{e^{2x}} = x\cancel{e^{2x}}$

3 for part sol. $\quad \therefore d_2 = 0 \;\&\; d_1 = 1$

$\therefore y = e^{2x}(C\cos x + D\sin x + x)$ 1 for gen sol.

Had the RHS been either $\cos x\, e^{2x}$ or $\sin x\, e^{2x}$, we would need the trial,

$y_p = (d_1 \cos x + d_2 \sin x)\, x e^{2x}$ 1