

Math 256. Sample Midterm exam.

No formula sheet, books or calculators!

Part I

Circle what you think is the correct answer. +3 for a correct answer, -1 for a wrong answer, 0 for no answer.

1. The ODE $y' + xy = xe^{x^2/2}$, $y(0) = 1$, has the solution,

- (a) $\frac{e^{x^2/2} + e^{-x^2/2}}{2}$ (b) $\frac{e^{x^2/2} - e^{-x^2/2}}{2}$ (c) $e^{x^2/2}$
(d) e^x (e) *None of the above.*

2. The ODE $y' = y^2 f(x)$, has the solution,

- (a) $[C + \int^x f(\tilde{x})d\tilde{x}]^{-1}$ (b) $[C - \int^x f(\tilde{x})d\tilde{x}]^{-1}$ (c) $[C + \int^x f(\tilde{x})d\tilde{x}]^{-2}$
(d) $[C - \int^x f(\tilde{x})d\tilde{x}]^{-2}$ (e) *None of the above,*

where C is a constant.

3. The ODE $y'' + 6y' + 25y = 0$, has the solution,

- (a) $Ae^{(3+4i)x} + Be^{-(3+4i)x}$ (b) $e^{-3x}(A \cos 4x + B \sin 4x)$ (c) $Ae^{3x} \cos(3x + B)$
(d) $Ae^{-3x} \cos(3x + B)$ (e) *None of the above,*

where A and B are constants.

4. The ODE $y'' - 3y' - 4y = -4x^2$, has the particular solution,

- (a) $x^2 - 3x/2 + 13/8$ (b) $x^2 - 3x/2 - 13/8$ (c) $x^2 + 3x/2 + 13/8$
(d) $x^2 + 3x/2 - 13/8$ (e) *None of the above.*

Part II

Answer in full (i.e. give as many arguments, explanations and steps as you think is needed for a normal person to understand your logic). Answer as much as you can; partial credit awarded.

1. Define the integrating factor for the first-order ODE, $y' + yp(x) = q(x)$. Find the integrating factor of the equation, $y' + y \ln x = x^{-x}$. If $y(1) = 0$, show that the solution is

$$y(x) = A(x^{-x}e^{x-1} - x^{-x}),$$

where A is a constant that you should determine. *Hint:* $\int \ln x \, dx = x \ln x - x$.

2. Find the solution to

$$2y'' + 2y' - 4y = 10 \cos x + 10 \sin x, \quad y(0) = y'(0) = 0.$$