**Math 256. Sample Midterm exam.**

No formula sheet, books or calculators!

## Part I

Circle what you think is the correct answer. +3 for a correct answer, −1 for a wrong answer, 0 for no answer.

**1**. The ODE $y' - yp(x) = 0$, has the solution,

$$(a)\quad CJ \qquad (b)\quad J + C \qquad (c)\quad J - C \qquad (d)\quad C/J \qquad (e)\quad None\ of\ the\ above,$$

where $C$ is a constant and $J = \exp[-\int^x p(\tilde{x})d\tilde{x}]$.

**2**. The ODE $y' + f(x)/y = 0$, has the solution,

$$(a)\quad \pm[C + 2\int^x f(\tilde{x})d\tilde{x}]^{1/2} \qquad (b)\quad \pm[C - 2\int^x f(\tilde{x})d\tilde{x}]^{1/2} \qquad (c)\quad \pm[C + \frac{1}{2}\int^x f(\tilde{x})d\tilde{x}]^2$$

$$(d)\quad \pm[C - \frac{1}{2}\int^x f(\tilde{x})d\tilde{x}]^2 \qquad (e)\quad None\ of\ the\ above,$$

where $C$ is a constant.

**3**. The ODE $y'' - 4y' + 5y = 0$, has the solution,

$$(a)\quad e^{2x}(A\cos x + B\sin x) \qquad (b)\quad e^{-2x}(A\cos x + B\sin x) \qquad (c)\quad Ae^{2x}\cos(2x + B)$$

$$(d)\quad Ae^x\cos(x + B) \qquad (e)\quad None\ of\ the\ above,$$

where $A$ and $B$ are constants.

**4**. The ODE $y'' + y' + 2y = 4x^2$, has the particular solution,

$$(a)\quad 2x^2 - 2x + 1 \qquad (b)\quad 2x^2 - 2x - 1 \qquad (c)\quad 2x^2 + 2x + 1$$

$$(d)\quad 2x^2 + 2x - 1 \qquad (e)\quad None\ of\ the\ above.$$

## Part II

Answer in full (i.e. give as many arguments, explanations and steps as you think is needed for a normal person to understand your logic). Answer as much as you can; partial credit awarded.

**1**. Define the integrating factor for the first-order ODE, $y' + yp(x) = q(x)$.
Hence $(1 - x^2)y' - xy = \sqrt{1 - x^2}(1 + x^2)^2$ with $y(0) = 0$.

**2**. Solve the ODE,

$$y'' - 4y' + 4y = e^{\lambda x}, \qquad y(0) = y'(0) = 0,$$

for (a) $\lambda \neq 2$, and (b) $\lambda = 2$.