MATH 257/316 Assignment 2
Supplementary Exercises - Not to be handed in.

Problem 1:
Consider the IVP (initial value problem):

$$(x-1)y'' - (x-3)y' - y = 0 , \qquad y(0) = 3 , \qquad y'(0) = 3 .$$

Determine a power series solution centred about $x_0 = 0$.
Rewrite your answer in terms of elementary functions.

Problem 2:
By using a power series representation centred about $x_0 = 0$ determine the general solution of

$$(x^2 + 1)y'' + \frac{7}{2}xy' + y = 0 .$$

Problem 3:
Substitute the power series $\sum_{n=0}^{\infty} a_n x^n$ into

$$y'' + x^2 y' + y = 0 ,$$

and determine a recursion relation describing the a_n's.
Compute a_2, a_3, a_4 and a_5 in terms of a_0 and a_1.

Problem 4:
Find all singular points of the following ODEs and determine whether each one is regular or irregular.

a) $\quad x^2 y'' + 3y' + 2xy = 0$

b) $\quad (x^2 - 1)^2 y'' + (x+1)y' + (x+2)y = 0$

c) $\quad \sin^2(x)y'' + \sin(\frac{x}{2})y' + 2y = 0$

d) $\quad x^2 \sinh(x)y'' + x^3 y' + \cosh(x)y = 0$

Problem 5:
Determine a lower bound for the radius of convergence of series solutions for the given ODE about the corresponding point x0. (You should not try to compute the series solution itself.)

a) $\quad (x^2 - 3)y'' + y' + \tan(x)y = 0, \ x_0 = 0$

b) $\quad (4x^2 + 9)y'' + x^3 y' + y = 0. \ x_0 = 2$