MATH 257/316 Assignment 6 (supplementary Exercises)
Due: Never

Problem 1: Fourier Series

a) **(B&D Sec 10.4 #7)** Sketch the graph of the even and odd extension of f of period $2L$, for the function f defined on $[0, L]$ as follows:

$$f(x) = \begin{cases} x \text{ for } 0 \leq x < 2 \\ 1 \text{ for } 2 \leq x < 3 \end{cases}$$

b) **(B&D Sec 10.4 #16)**Sketch the graph of the function to which the Fourier Sine Series expansion of period 4 of the following function converges:

$$f(x) = \begin{cases} x \text{ for } 0 \leq x < 1 \\ 1 \text{ for } 1 \leq x < 2 \end{cases}$$

c) Determine the Fourier series for the "triangular wave" defined by

$$f(x) = \begin{cases} -x \text{ for } -l \leq x < 0 \\ x \text{ for } 0 \leq x < l \end{cases}$$

Use this Fourier Series to show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

Note that evaluating the first three terms in the series we obtain the following approximation for π :

$$\pi \simeq \left(8(1 + \frac{1}{3^2} + \frac{1}{5^2})\right)^{1/2} = 3.0346$$

Now use the seies and Parseval's Theorem to show that:

$$\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^4}$$

Note that evaluating the first three terms in the series we obtain the following approximation for π :

$$\pi \simeq \left(96(1 + \frac{1}{3^4} + \frac{1}{5^4})\right)^{1/4} = 3.1410256,$$

which converges much more rapidly than the first series.

Problem 2: Steady state solutions ((B&D Sec 10.6 #1 & 7).

Find the steady-state solutions for the heat equation $u_t = \alpha^2 u_{xx}$ that satisfy each of the following boundary conditions:

a) $u(0, t) = 10$, $u(50, t) = 40$

b) $u_x(0, t) - u(0, t) = 0$, $u(L, t) = T$

Problem 3: Heat conduction with inhomogeneous boundary conditions (B&D Sec 10.6 #11).

Consider a rod of length 30 units for which $\alpha^2 = 1$. Suppose that the initial temperature distribution is given by $u(x, 0) = x(60 - x)/30$.and that the boundary conditions are $u(0, t) = 30$ and $u(30, t) = 0$.

a) Find the temperature in the rod as a function of position and time.

b) Does the temperature approach a steady-state as $t \to \infty$? If so, what is the steady-state solution?

b) Plot u versus t for $x = 12$. Observe that the temperature decreases initially and then increases, and then finally decreases to reach a steady state value. Explain physically why this behaviour occurs at this point.