Int. J. Rock Mech. & Min. Sci. Vol. 34, No. 3-4, 1997 ISSN 0148-9062
To cite this paper: *Int. J. Rock Mech. & Min. Sci.* **34:3–4**, Paper No. 216

THE USE OF A MULTIPOLE EXPANSION TECHNIQUE TO ANALYSE LARGE SCALE FRACTURE PROCESSES AND SEISMIC RECURRENCE EFFECTS IN DEEP LEVEL MINES

J. A. L. Napier[1]; A. P. Peirce[2]

[1] CSIR Division of Mining Technology, Johannesburg, South Africa

[2] Department of Mathematics, University of British Columbia, Vancouver, Canada

ABSTRACT

An efficient solution technique for the analysis of multiple interacting displacement discontinuities is described together with the implementation of a viscoplastic relaxation model for the simulation of observed time dependent seismic activity in deep level gold mines. The method is illustrated in relation to the mining of a parallel sided panel with attendant mobilization of fractures in the surrounding rock. A comparison is made between the case where mining is advanced continually by a constant increment every day and the case where mining is interrupted every seventh day corresponding to a weekend break. Some possible methods of analysing the seismic recurrence cycles that are induced by the mining are presented. For the simple cases studied it is found that there is no apparent correlation between successive seismic energy release peaks. It appears that slightly more seismic energy is released in the continuous mining cycle than in the case where the mining is interrupted every seventh day.

KEYWORDS

DISCONTINUITY CREEP • MINE SEISMICITY • MULTIPOLE METHODS • SEISMIC RECURRENCE • VISCOPLASTICITY

INTRODUCTION

The occurrence of time dependent migration of seismic activity near deep level gold mine workings has been qualitatively observed for many years but has, more recently, been extensively monitored by high quality seismic networks, Mendecki 1993. Seismic migration effects have also been noted in association with rock mass preconditioning experiments where the seismic activity is observed to move from preconditioned mining faces towards adjacent areas that are more highly stressed, Lightfoot *et al.* 1996. The effects of time dependent movements and damage processes near deep level mine excavations have an important bearing on a number of practical mine design issues. These include slow or sudden slip on fault planes and their interaction with mine panels as well as considerations of the consequences of varying the stope face advance rate. A high rate of face advance may be desirable from the viewpoint of concentrated mining logistic efficiency but may also attract greater levels of seismic activity. Alternatively, excessively slow rates of face advance may lead to additional damage to the rock and to support elements and thereby additional fallout in the excavation. A related question concerns the effect of introducing single or double shift daily mining cycles and the consequences of mining throughout the week or interrupting the mining every weekend for one day. In addition, it is important to be able to quantify the desirability of introducing a single blasting time in a mine in order to avoid undesirable interference between mine panels and to ensure that the peak seismicity induced by the face blasts is confined to periods when the mine is vacated, Riemer 1997.

From these considerations it is clearly necessary to be able to quantify the mechanistic processes that accompany large scale seismic activity and to represent these processes in some numerical form that can be used for routine mine design studies. In previous work by Napier , Malan 1997, it has been suggested that an appropriate numerical approach is to represent damaged regions in the rock mass by explicit assemblies of interacting cracks. Each crack can be assumed to "creep" according to a postulated rheological constitutive law and the mutual interaction between the cracks can be followed as a function of time. This approach is very similar to the simulation of earthquake recurrence cycles in which the fault is modelled as a cellular grid of interacting slip elements, Rice 1993, Ben-Zion , Rice 1993. In principle, any suitable numerical technique can be employed to analyse the interaction between the slip elements; the most appropriate tools for this purpose are the Distinct Element Method, Cundall 1988, or the displacement discontinuity method, Crouch , Starfield 1983. Both approaches involve long run times if large numbers of cracks are to be analysed. In addition, the displacement discontinuity method requires explicit influence functions to be computed between every pair of interacting elements which cover the crack surfaces. A novel method of reducing the computational effort in resolving this interaction, termed the multipole method, has been developed by Peirce , Napier 1995. The present paper gives a brief review of the multipole method and describes its extension to include time dependent discontinuity creep. The multipole creep model is then applied to a particular problem of interest in which a narrow width stope is mined according to a cycle of one blast per day and, by contrast, a cycle in which no blast is taken on every seventh day. These analyses are carried out for panels which are advanced from initially small spans. It must be emphasized that the multipole method currently being used has only been developed for two dimensional plane problems. A future challenge is to develop an equivalent model for three dimensional analyses.

THE MULTIPOLE METHOD

A detailed description of the multipole method for the solution of large scale interacting crack problems has been given by Peirce , Napier 1995. A short summary of the approach is presented here. The stress tensor τ_{kl} at a given point P in an isotropic elastic medium which is cut by discontinuity surfaces S_d can be shown to be given by

$$\tau_{kl}(P) = -\int_{S_d} \Gamma_{ijkl}(P,Q)\Delta_{ij}(Q)dS_Q \tag{1}$$

where Γ_{ijkl} is a fourth order influence tensor given by the expression

$$\Gamma_{ijkl} = \frac{G}{4\pi(1-\nu)}[\psi_{,ijkl} - 2\nu(\delta_{kl}\psi_{,ij} + \delta_{ij}\psi_{,kl})_{,mm} - (1-\nu)(\delta_{ik}\psi_{,jk} + \delta_{il}\psi_{,jk} + \delta_{jk}\psi_{,il} + \delta_{jl}\psi_{,ik})_{,mm}] \tag{2}$$

Commas denote differentiation with respect to the components p_i of the field point P and repeated subscripts imply summation over the appropriate space dimension (two in the present case). G is the shear modulus and ν is the Poisson's ratio of the material. δ_{ij} is the Kronecker delta and ψ is a two

ISSN 0148-9062

To cite this paper: Int. J. Rock Mech. & Min. Sci. 34:3-4, paper No. 216.

dimensional biharmonic function which depends on the distance $r = |P - Q|$ and is given by

$$\psi(r) = \tfrac{1}{2}[r^2 - r^2 \log r^2] \tag{3}$$

The discontinuity tensor $\Delta_{ij}(Q)$ in equation (1) is expressed in terms of the displacement discontinuity components $D_i(Q)$ and the normal components $n_i(Q)$ at point Q of the discontinuity surface S_d by

$$\Delta_{ij}(Q) = \tfrac{1}{2}[D_i(Q)n_j(Q) + D_j(Q)n_i(Q)] \tag{4}$$

It should be noted that the components D_i and n_j are in global co-ordinates. Designating the co-ordinates of points P and Q by p_i and q_i respectively, the biharmonic potential ψ can be written as

$$\psi(r) = \psi(p_i - q_i^\circ + q_i^\circ - q_i) \tag{5}$$

where q°_i are the co-ordinates of a point Q° which is located on a regular rectangular mesh. It is assumed that in a local region the points Q on the discontinuity surface S_d are "close" to Q° in the sense that $|Q^\circ - Q| \ll |P - Q^\circ|$. If this is the case then equation (5) can be expanded in a Taylor series of the form

$$\psi(r) = \psi^\circ + (q_i^\circ - q_i)\psi^\circ_{,i} + \tfrac{1}{2}(q_i^\circ - q_i)(q_j^\circ - q_j)\psi^\circ_{,ij} + \ldots \tag{6}$$

where $\psi^\circ_{,i} \equiv \partial / \partial p_i[\psi(r^\circ)]$ and $r^\circ = |P - Q^\circ|$. Since the derivatives in equation (2) are with respect to the field point P, the evaluation of equation (1) depends on integrating expressions of the form

$$I_{rs} = \int \psi(r)\Delta_{rs}(Q)dS_Q \tag{7}$$

Substituting the expansion (6) into equation (7) yields

$$I_{rs} = \psi(r^\circ)M^\circ_{rs} + \psi_{,i}(r^\circ)M^i_{rs} + \psi_{,ij}(r^\circ)M^{ij}_{rs} + \ldots \tag{8}$$

where the so-called multipole strain moments $M^{ij\cdots}_{rs}$ are defined by

$$\begin{aligned} M^\circ_{rs} &= \int \Delta_{rs}(Q)dS_Q \\ M^i_{rs} &= \int (q_i^\circ - q_i)\Delta_{rs}(Q)dS_Q \\ M^{ij}_{rs} &= \tfrac{1}{2}\int (q_i^\circ - q_i)(q_j^\circ - q_j)\Delta_{rs}(Q)dS_Q \quad \text{etc.} \end{aligned} \tag{9}$$

It can be seen from equations (8) and (9) that the strain moments depend on the source point Q and on the functional form of the discontinuity tensor $\Delta_{rs}(Q)$. In the present case this is chosen to be a linear variation function over piecewise flat surface elements although it is a simple matter to include higher order function variations or even curved elements into the multipole methodology. The function $\psi(r^\circ)$ and its derivatives depends only on the field point P and the grid point Q°. If the stress components τ_{kl} are evaluated at points P° belonging to the same regular mesh as the points Q°, it becomes possible to implement a Fast Fourier Transform (FFT) scheme to evaluate the influences at all points of the mesh in

ISSN 0148-9062

a time that is proportional to $N \log N$ where N is the number of points in the mesh. In order to evaluate stresses at an arbitrary receiving position P that is not on the mesh, an interpolation scheme is used to evaluate the stresses at P in terms of the values available at surrounding grid points P_i°. In addition, in order to limit the stress gradients so that interpolation is possible without compromising accuracy, it has been found necessary to evaluate the stress components using only multipole strains that are outside a given pad distance from the receiving point. Remaining influences are computed by direct application of equation (1) within the pad regions. This approach allows for large scale irregular geometries to be analysed efficiently in terms of remote influences and also retains sufficient accuracy with respect to local details of the discontinuity geometry.

VISCOPLASTIC MODEL

The exact nature of time dependent deformations in hard rock mines is poorly understood. It is generally accepted that the rate of creep of intact rock samples is much slower than that which is consistent with the observations of time dependent movements that are made in deep level mines, Malan *et al.* 1997. Consequently, it is assumed that the rheological components of deformation are associated with creep on existing or mining induced discontinuities. These creep movements may be caused by time dependent micro breakage of asperities which in turn may be affected by some form of chemical corrosion process induced by ambient moisture or blast gases, Atkinson 1984. Alternatively, discontinuities which have infilling gouge material may creep according to the viscoplastic behaviour of the gouge. At present it is not clear which of these mechanisms is predominant and the rheological behaviour is presumably controlled by local geological structures and by the exposure time of excavations to weathering processes. In the case of time dependent asperity breakage, it is convenient to postulate that the creep rate is proportional to the shear driving stress acting on the discontinuity, Wesson 1988. This can be expressed in terms of the rate of change of the shear displacement discontinuity D_s as

$$dD_s / dt = e\kappa_s \tau_e \tag{10}$$

where t is the time and where

$$\tau_e = |\tau| - \mu\sigma_n - S_{\circ} \tag{11}$$

is the net shear driving stress. τ is the local shear stress acting across the discontinuity, σ_n is the normal stress (compressive positive) and μ is the coefficient of friction. e is an indicator variable that depends on the sign of the shear stress τ. κ_s is a material constant that can be thought of as a "surface" fluidity. $S_{\circ}$ is the discontinuity cohesion which may itself be time dependent or dependent on the shear slip D_s. More elaborate models in which the friction μ is dependent on time or on the joint slip velocity can be considered (Ruina 1983). No slip occurs if τ_e is negative. The parameter κ_s in equation (10) depends on local asperity breakage and corrosion phenomena on the discontinuity surface and has units $Pa^{-1}\ m\ s^{-1}$. Spottiswoode , Malan 1997 have shown that the analysis of time dependent deformation using equation (10) yields a strain rate that is in good agreement with the rate of seismicity over scales of hours. Additional research is needed to clarify and substantiate the application of equation (10) or alternative formulations which include explicit reference to the discontinuity thickness.

A particular problem is specified by defining a set of pre-existing or potential discontinuity sites in the region of interest and is solved in a series of time steps. All discontinuities are approximated by

ISSN 0148-9062

appropriately positioned flat elements and each element is designated as being “instantaneous” (I) or “viscoplastic” (**V**). An “instantaneous” element is understood to be such that the displacement discontinuity components are computed to satisfy instantaneously the current stress boundary conditions acting on the element whereas a “viscoplastic” element is such that the shear displacement discontinuity is relaxed according to equation (10) in each time step. At the start of each time step a search is carried out to determine whether any unmobilized elements should be activated according to the current stress state and depending on the strength of the element. The strength is expressed in terms of local cohesion, friction and tension cutoff parameters pertaining to each crack. Following the selection of elements, the stress interaction between all **I** elements is evaluated and the equilibrium displacement discontinuity components on each “instantaneous” element is determined. In this process, the influence of all slipped viscoplastic elements is considered to remain fixed. After this initial solution step, all viscoplastic elements are considered and the slip on each element is updated according to the relaxation law represented by equation (10). At present a first order Euler scheme is used to integrate equation (10). At the end of the time step all elements are reviewed to determine if a transition from **V** to **I** or **I** to **V** is to be made. For example, if a viscoplastic element opens during the time step, it is subsequently considered to behave as an “instantaneous” element. The time stepping procedure is described in detail in Napier , Malan 1997.

SEISMIC ENERGY RELEASE COMPUTATION

In the analysis of mining problems, it is convenient to define the gravitational body forces and far field tectonic forces as “loading” forces. In the course of mining, discontinuities such as faults, joints and mining induced fractures are mobilized and new excavation surfaces are created. Let the incremental work that is done by the loading forces during a given time interval be designated as ΔW_L. If the rockmass is considered to behave elastically, apart from any dissipation processes that occur on the discontinuity surfaces S_d, then conservation of energy requires that

$$\Delta W_L = \Delta U + \Delta W_A \tag{12}$$

where ΔU is the change in internal strain energy and ΔW_A represents the kinetic energy changes and energy dissipated by frictional sliding on discontinuities. It can be shown (Napier 1991) that ΔW_A can be computed directly from the traction components $T_i(Q)$ and the discontinuity components $D_i(Q)$ according to

$$\Delta W_A = -\tfrac{1}{2}\int_{S_d}[T_i^t(Q) + T_i^p(Q)][D_i^t(Q) - D_i^p(Q)]dS_Q \tag{13}$$

The superscripts p and t refer to the start and end of the current time step. The level of kinetic or seismic energy ΔW_S that is dissipated during the transition from state p to state t can be expressed as

$$\Delta W_S = \Delta W_A - \Delta W_D \tag{14}$$

where ΔW_D is the energy dissipated by frictional sliding during the time step. This is given by

$$\Delta W_D = \int_{S_d} \tan\phi \overline{T}_n(Q) \left| D_s^t(Q) - D_s^p(Q) \right| dS_Q \tag{15}$$

where $\overline{W}_n(Q)$ is the average normal stress and $D^p{}_s(Q)$ and $D^t{}_s(Q)$ are the local shear components of the displacement discontinuity at the start and end of the time step respectively. In this paper, it is assumed that ΔW_S can be used as a measure to assess the impact of mining at different rates or to evaluate seismic recurrence cycles in relation to mining step advances.

APPLICATION TO MINING PROBLEMS

The viscoplastic multipole model is illustrated by considering the initial mining of a parallel sided panel in incremental steps. Estimates of the seismic energy released during each time step are made according to equation (14). The panel is assumed to be horizontal and is approximated as a narrow crack whose span is enlarged in 40 mining steps of one metre per step. The panel is surrounded by a rectangular region covered by a random mesh of potential crack segments. For convenience the mesh is generated as a Delaunay triangulation. The total length of mesh segments is 5912m and is confined to vertical section 60m wide and 30m high as shown in Figure 1.

The average length of the mesh segments is 1,12m with a standard deviation of 0,28m. The mesh segments above the panel horizon (region H in Figure 1) are assumed to be stronger than the segments below the panel (region F). The specific properties are summarized in Table 1. The lines marked A in Figure 1 depict the highly stressed region where new fractures are formed in response to the last face advance step. The regions B and C above and below the stope have low stresses and are pervasively fractured by previous mining steps.

The far field stress is assumed to have a vertical component of 60 MPa and a horizontal component of 30 MPa. The intact elastic material has a Young's modulus of 70000 MPa and a Poisson's ratio of 0,2. It is assumed that the segments of the mesh behave in a viscoplastic manner in shear with a “fluidity” of κ_S = 0,0005 MPa^{-1} m day^{-1}. Figure 2a shows the estimated recurrence of seismic energy release when 40 mining steps are carried out at a face advance rate of one metre per day. The time step interval is 0,1 days implying that 400 successive problems are solved. The total run time was approximately four days on a PC equipped with a 200 MHz Pentium processor and up to 1500 elements were mobilized. Following each face advance step there is a sharp increase in released energy followed generally by a monotonic decay in activity until the next face advance increment. The sizes of the initial five peaks in Figure 2a are seen to increase steadily but thereafter become progressively more erratic as the mining face advances. Small sub-peaks of activity are also observed to occur between successive advance steps as the panel span is increased. This behaviour is qualitatively very similar to that observed in deep level South African gold mines, McGarr 1971, Heunis 1980. It can be seen also from Figure 2a that the minimum level of activity displays an increasing trend as the span of the panel is increased. The corresponding length of segments that are mobilized during each time step are shown in Figure 2b and also displays an increasing trend as mining progresses.

A particular question of considerable importance is whether the level of seismic activity accompanying successive mining steps can be predicted in advance. The results shown in Figure 2a suggest that some form of time series analysis could be used to expose the existence or absence of predictable trends. A first step in this analysis is to consider the autocorrelation between the successive seismic energy release

ISSN 0148-9062

peaks in Figure 2a. Let the peak value at time t be designated as x_t then the autocorrelation coefficient, c_k , at lag k can be estimated from (Chatfield 1980)

$$c_k = \sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x}) / N \qquad (16)$$

where N are the total number of data points and $\bar{x}$ is the mean value of the series. An estimate of the autocorrelation function is given by the discrete values $r_k = c_k / c_0$. The autocorrelation numbers r_k are computed for the last 31 peak values of seismic energy release shown in Figure 2a. (The first 9 peaks are ignored as they exhibit an initial build up of activity). The results are shown in Table 2.

The approximate 95% confidence limits are $\pm 2/\sqrt{N}$ which in the present case are equal to ±36 and strongly suggest that the successive peaks are uncorrelated. Further investigation is however required to determine whether any longer term correlations can be established and also how to link the simulated seismic activity to actual observations.

A practical issue of some importance is the effect on seismic activity of a break in the mining cycle every weekend. At present many mines in South Africa do not blast on Sundays. It may be considered desirable, for economic efficiency, to pursue a seven day mining cycle in which mining is not interrupted every seventh day. The analysis shown in Figure 2a was repeated with 20 relaxation time steps being introduced after every six face advance steps to simulate the weekend break. The results are shown in Figure 3. Again it is clear that the general level of activity increases steadily as the panel span is increased but no clear correlation seems to exist between successive energy release peaks. In particular, the peak energy release on the first advance step after each weekend appears to be randomly distributed. The cumulative energy released, at the end of each face advance step, is compared in Figure 4 for the two cases of continuous mining (filled squares) and mining with a weekend break (open triangles). Over the first 30 mining steps there seems to be little difference although after step 30 slightly more seismic activity appears to be associated with the continuous mining cycle. It is apparent that a more extensive series of mining steps should be analysed.

It is also interesting to consider whether, in each week, the seismic activity increases during the week as is sometimes observed in South African deep level gold mines (McGarr 1971). The successive mean values of the peak seismic energy release shown in Figure 3 are quoted in Table 3 and do suggest that the seismic activity increases towards the end of each week. However, it must be recalled that in this case the increasing span of the panel tends to impose a trend of increasing seismic activity over the simulated period of seven weeks. The analysis should be repeated for the case of a "mature" stope of wide span where total closure between the roof and the floor of the excavation is established in the back area.

CONCLUSIONS

It has been shown that large scale two dimensional problems involving interacting discontinuities, which include viscoplastic time relaxation behaviour, can be analysed effectively using a multipole expansion technique. The solution framework also allows seismic energy release trends associated with incremental mining steps to be simulated. The method is illustrated by analysing the initial mining of a parallel sided panel and comparing the energy release patterns if the mining is carried out on a continuous cycle seven days a week or if mining is interrupted every seventh day. It is found that slightly more seismic activity is associated with the seven day cycle. In the present simplistic analysis it is also found that no correlation

ISSN 0148-9062

exists between successive energy release peaks which, in this instance, precludes establishing any predictive models for seismic recurrence. However, in the case of mining with a break every seventh day, the seismic activity appeared, on average, to increase from the first to the last day in each week in agreement with previously observed trends.

The results presented here serve to illustrate a number of interesting new options for planning deep level mine layout extraction sequences. These include the potential to determine the optimum mining rate of extraction and the possibility of including explicit mechanistic representations of time dependent seismicity and seismic migration effects for general hazard analysis. Further work is required to link these models to actual observations and to establish a better understanding of discontinuity rheological deformation mechanisms. It is equally important to consider the development of three dimensional analysis techniques.

ACKNOWLEDGEMENTS

The authors wish to acknowledge that this work was carried out as part of the Mine Health and Safety research programme supported by the SIMRAC committee of the Department of Mineral and Energy Affairs of South Africa. We acknowledge a number of insightful comments from our colleagues Dr S M Spottiswoode, Mr M W Hildyard and Dr E J Sellers and very helpful discussions with Mr K L Riemer of Goldfields of South Africa.

FIGURES

Paper 216, Figure 1.

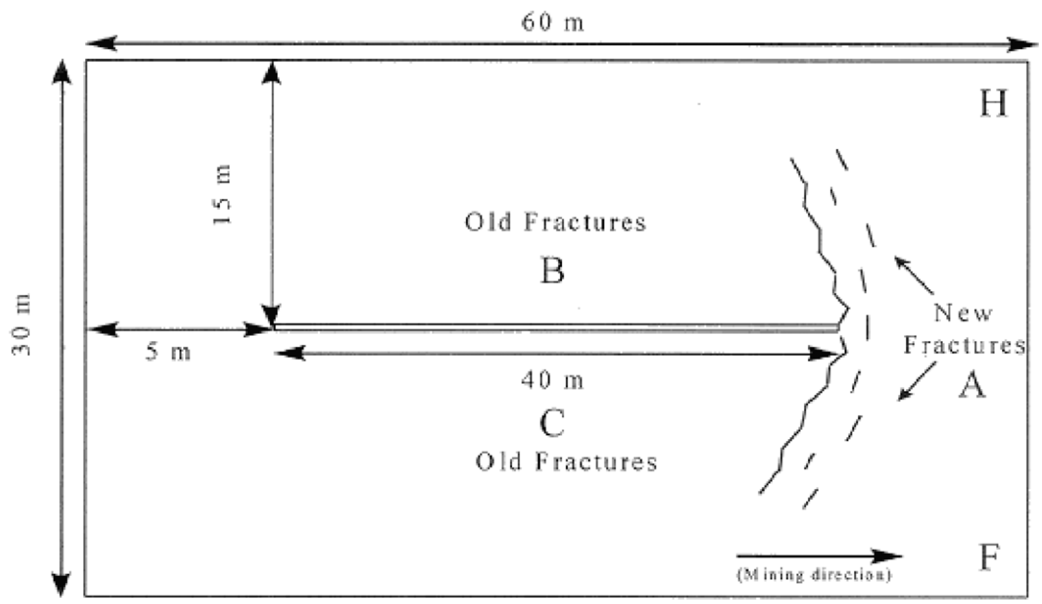

Figure 1. Illustrative mining problem of a parallel sided panel.

ISSN 0148-9062

Paper 216, Figure 2a.

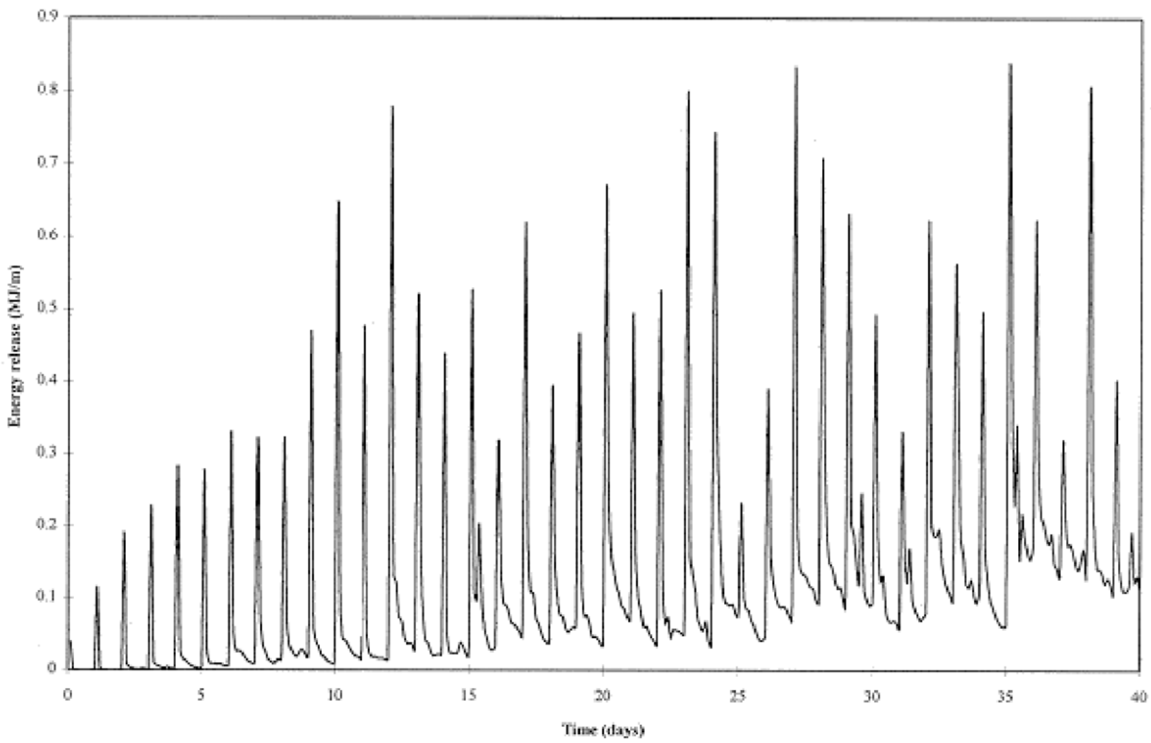

Figure 2a. Energy release per time step - mining seven days per week

ISSN 0148-9062

Paper 216, Figure 2b.

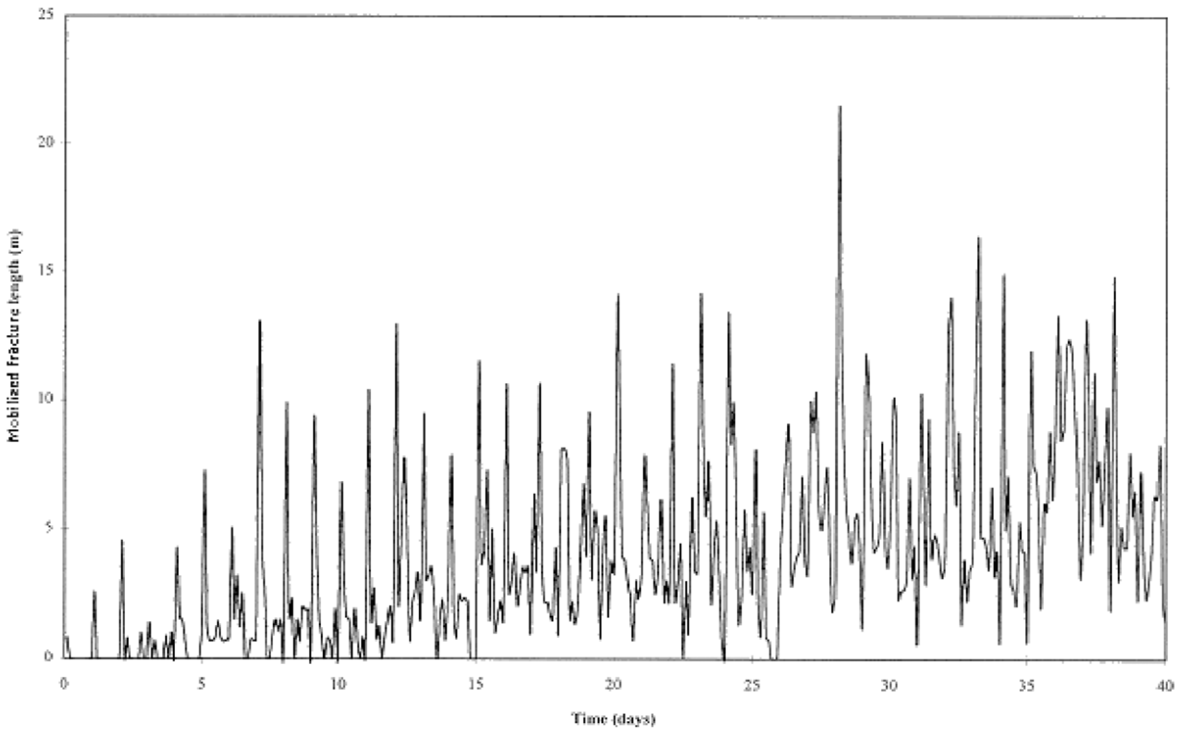

Figure 2b. Mobilized fracture length per time step - mining seven days per week

ISSN 0148-9062

Paper 216, Figure 3.

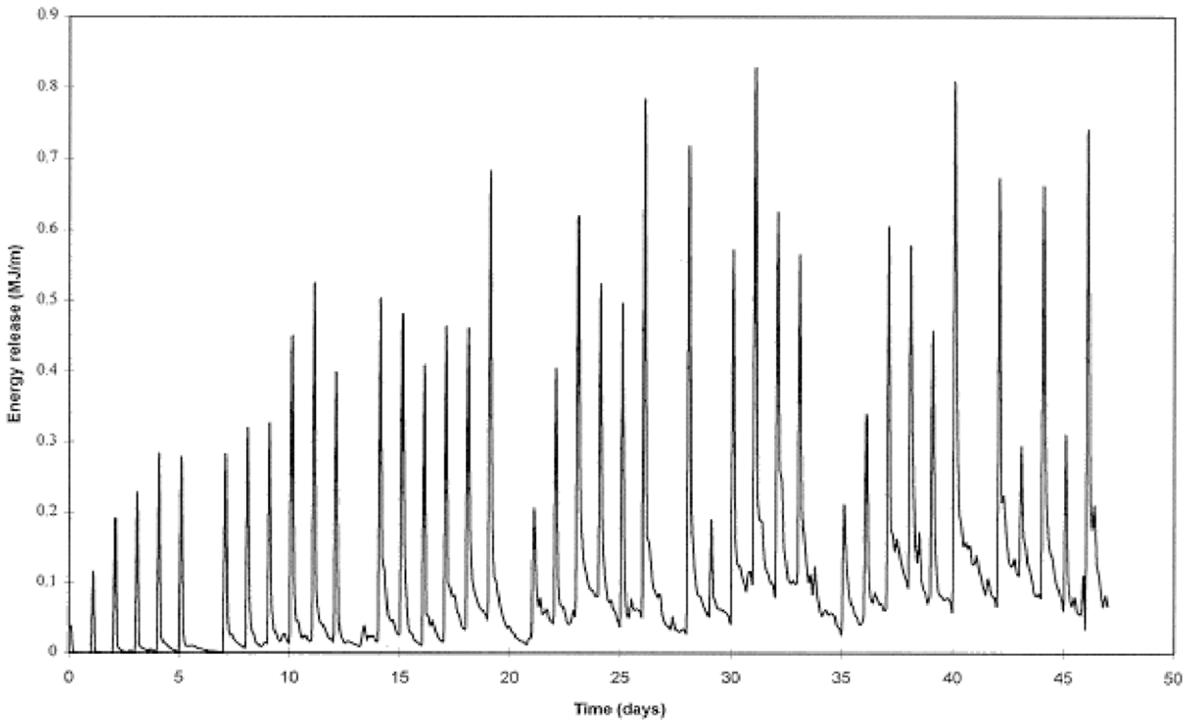

Figure 3. Energy release per time step - no mining every seventh day

ISSN 0148-9062

Paper 216, Figure 4.

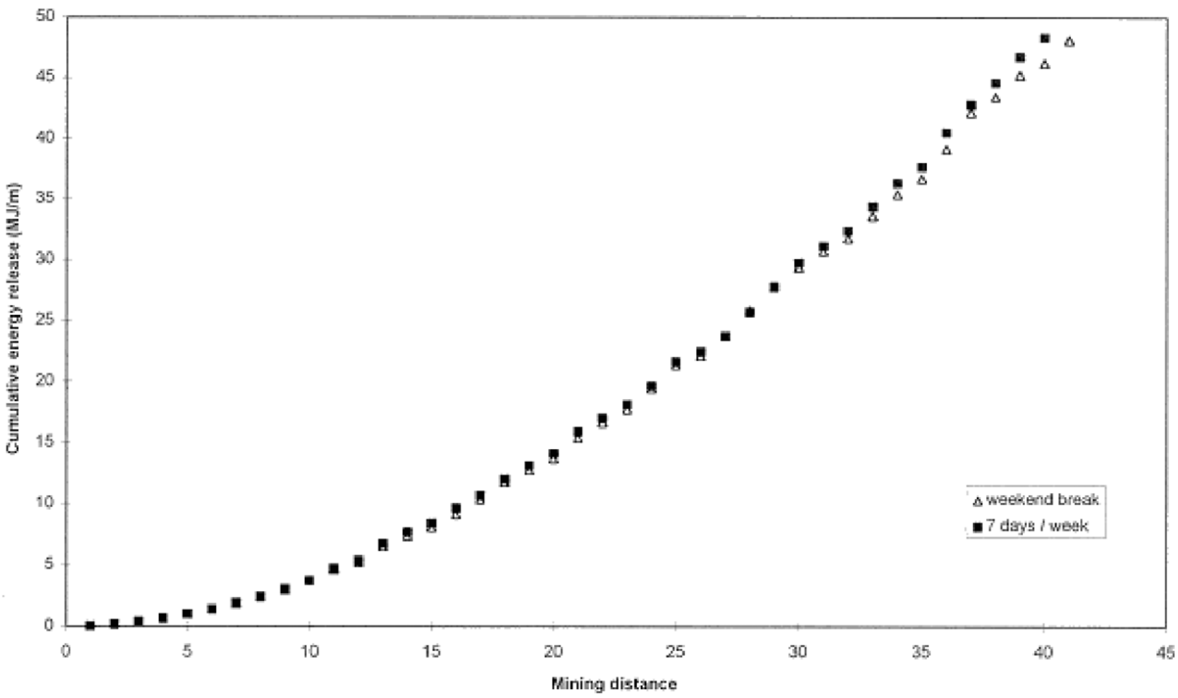

Figure 4. Comparison of cumulative energy released for continuous mining and mining with a break every seventh day

TABLES

Paper 216, TABLE 1.

TABLE 1 Material properties of H and F regions in illustrative mining problem.

Region	Cohesion (MPa)	Friction (degrees)	Mobilized Cohesion (MPa)	Mobilized Friction (degrees)	Tension Cutoff (MPa)
H	25	45	0	30	10
F	10	45	0	30	5

ISSN 0148-9062

Paper 216, TABLE 2.

TABLE 2 Autocorrelation of peak energy release rates

Lag (k)	0	1	2	3	4
ACF (r_k)	1,0	-0,163	-0,311	0,092	-0,188

Paper 216, TABLE 3.

TABLE 3 Peak energy release averaged for each day of the week

DAY	Mon	Tues	Wed	Thu	Fri	Sat
Average Peak Energy	0,375	0,307	0,483	0,483	0,512	0,586

References

References

Atkinson B.K. 1984. Subcritical crack growth in geological materials. *J. Geophys. Res.,* **89,** 4077–4114.

Ben-Zion Y., Rice J. R. 1993. Earthquake failure sequences along a cellular fault zone in a three-dimensional elastic solid containing asperity and nonasperity regions. *J. Geophys. Res.,* **98,** 14109–14131.

Chatfield M. 1980. *The Analysis of Time Series: An Introduction,* Chapman and Hall, London.

Crouch S.L., Starfield A.M. 1983. *Boundary Element Methods in Solid Mechanics,* George Allen & Unwin, London.

Cundall P.A. 1988. Formulation of a three-dimensional distinct element model - Part I. A scheme to detect and represent contacts in a system composed of many polyhedral blocks. *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.,* **25,** 107–116.

Heunis R. 1980. The development of rock-burst control strategies for South African gold mines. *J. S. Afr. Inst. Min. Metall.,* **80,** 139–150.

Lightfoot N., Goldbach O.D., Kullmann D.H., Toper A.Z. 1996. Rockburst control in the South African deep level gold mining industry. In: M. Aubertin, F. Hassani and H. Mitri (eds.) *Proc. 2nd. North American Rock Mechanics Symposium: NARMS'96,* 295–303.

Malan D.F., Spottiswoode S.M. 1997. Time-dependent fracture zone behaviour and seismicity surrounding deep level stoping operations. Accepted for publication in: S.J. Gibowicz (ed.) *Proc. 4th Int. Symp. Rockbursts and Seismicity in mines,* Krakow, Poland.

Malan D.F., Vogler U.W., Drescher K. 1997. Time-dependent behaviour of hard rock in deep level gold

ISSN 0148-9062

mines. Submitted to *J. S. Afr. Inst. Min. Metall.*

Mendecki A.J. 1993. Real time quantitative seismology in mines. Keynote Lecture. In: R.P. Young (ed.) *Proc. 3rd Int. Symp. Rockbursts and Seismicity in mines,* Balkema, 287–295.

Napier J.A.L. 1991. Energy changes in a rockmass containing multiple discontinuities. *J. S. Afr. Inst. Min. Metall.,* **91,** 145–157.

Napier J.A.L., Malan D.F. 1997. A viscoplastic discontinuum model of time-dependent fracture and seismicity effects in brittle rock. Paper submitted to *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*

Peirce A.P., Napier J.A.L. 1995. A spectral multipole method for efficient solution of large-scale boundary element models in elastostatics. *Int. j. numer. methods eng.,* **38,** 4009–4034.

Perzyna P. 1966. Fundamental problems in viscoplasticity. *Adv. Appl. Mech.,* **9,** 243–377.

Riemer K.L. 1997. Personal communication.

Rice J.R. 1993. Spatio-temporal complexity of slip on a fault. *J. Geophys. Res.,* **98,** 9885–9907.

Wesson R.L. 1988. Dynamics of fault creep. *J. Geophys. Res.,* **93,** 8929–8951.

ISSN 0148-9062