Hydraulic Fracture: multiscale processes and moving interfaces

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Nanoscale Material Interfaces:
Experiment, Theory and Simulation

Singapore

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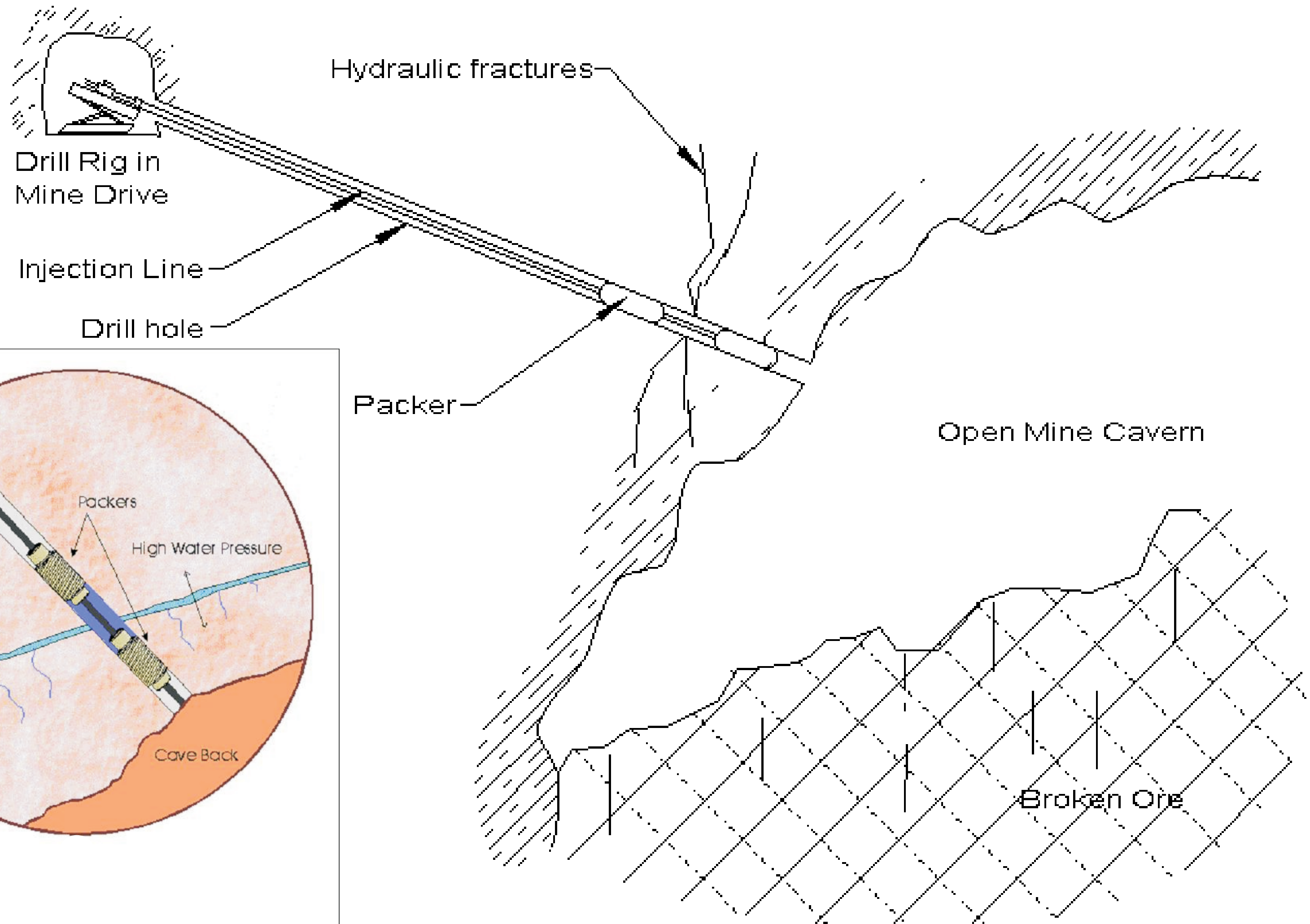
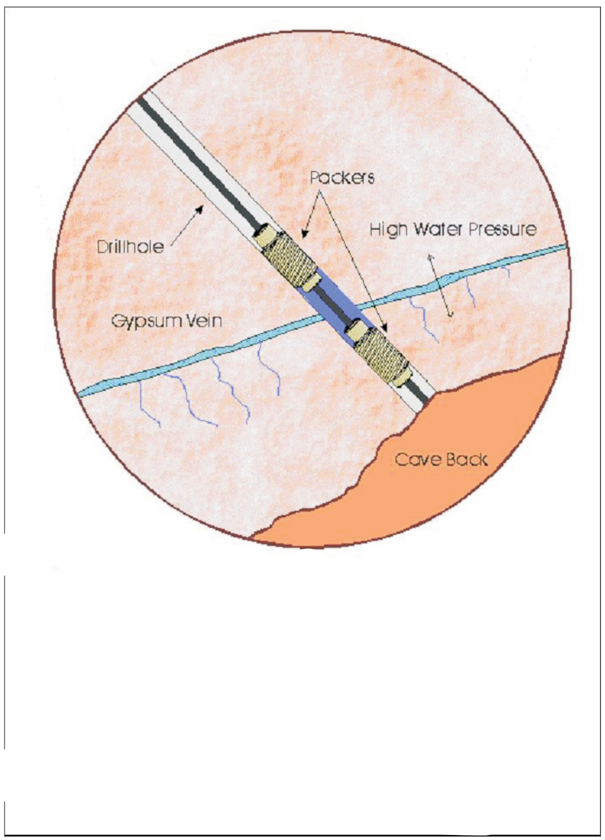
Outline

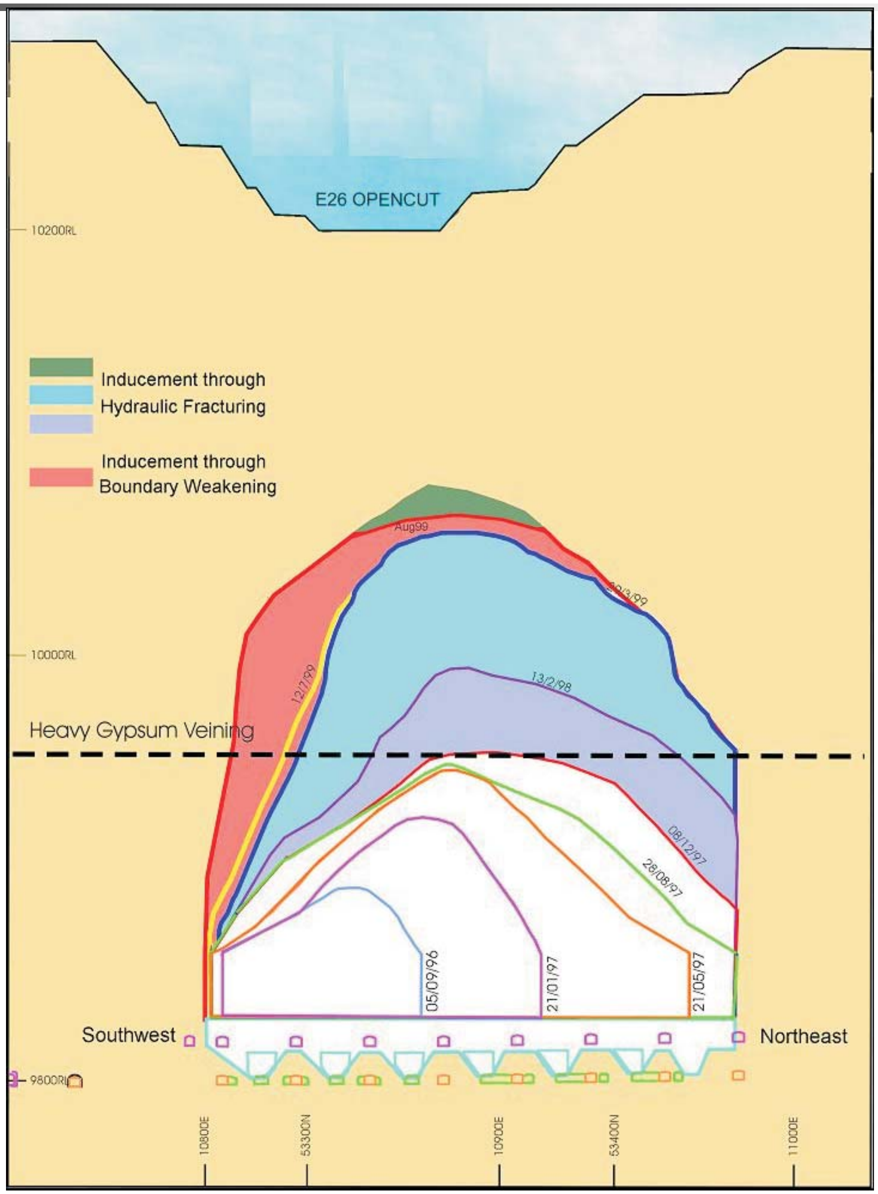
- What is a hydraulic fracture?
- Scaling and special solutions for 1D models
- Numerical modeling for 2-3D problems
- Conclusions

HF Examples - block caving

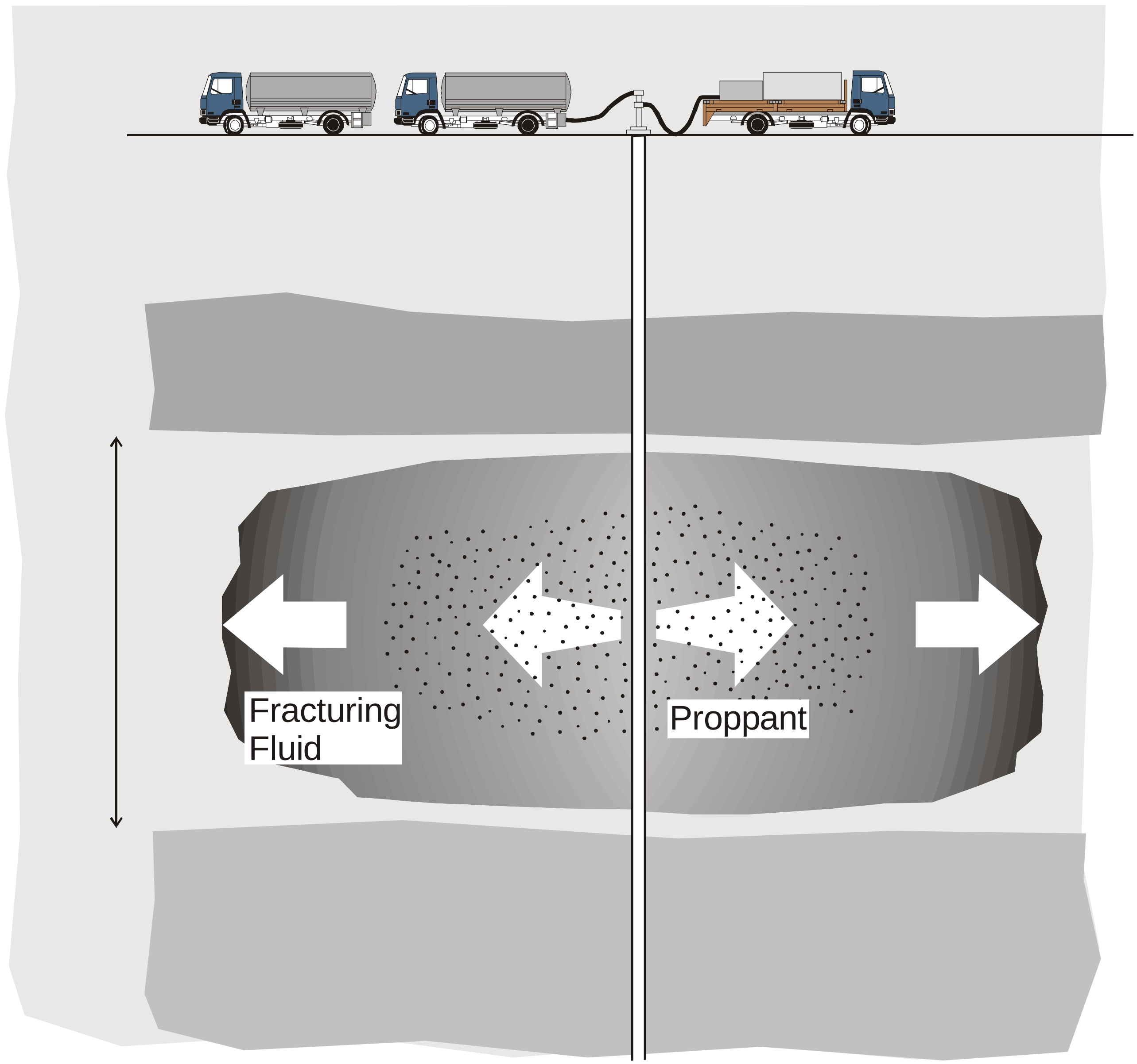




HF Example – caving (Jeffrey, CSIRO)

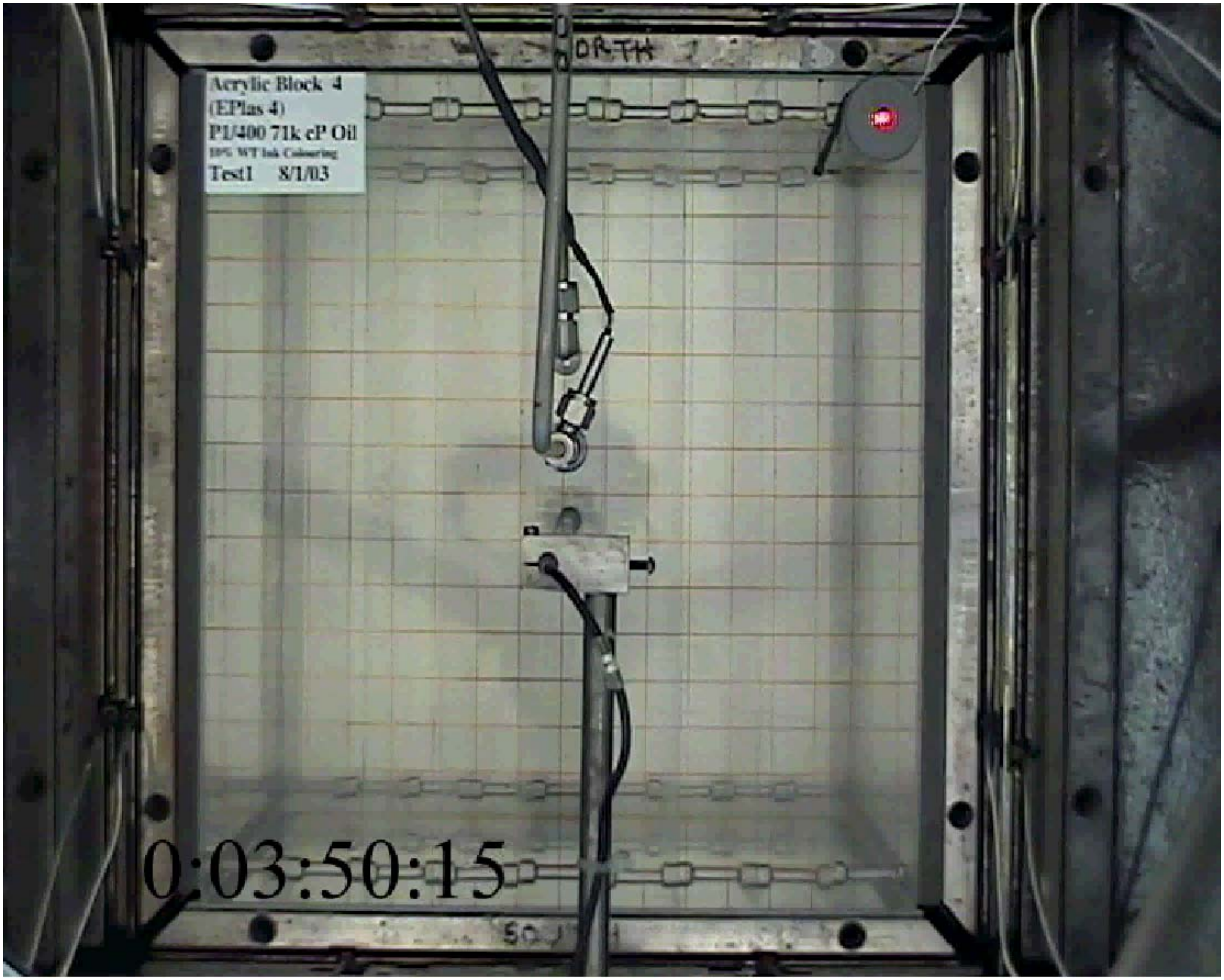


HF Examples – well stimulation



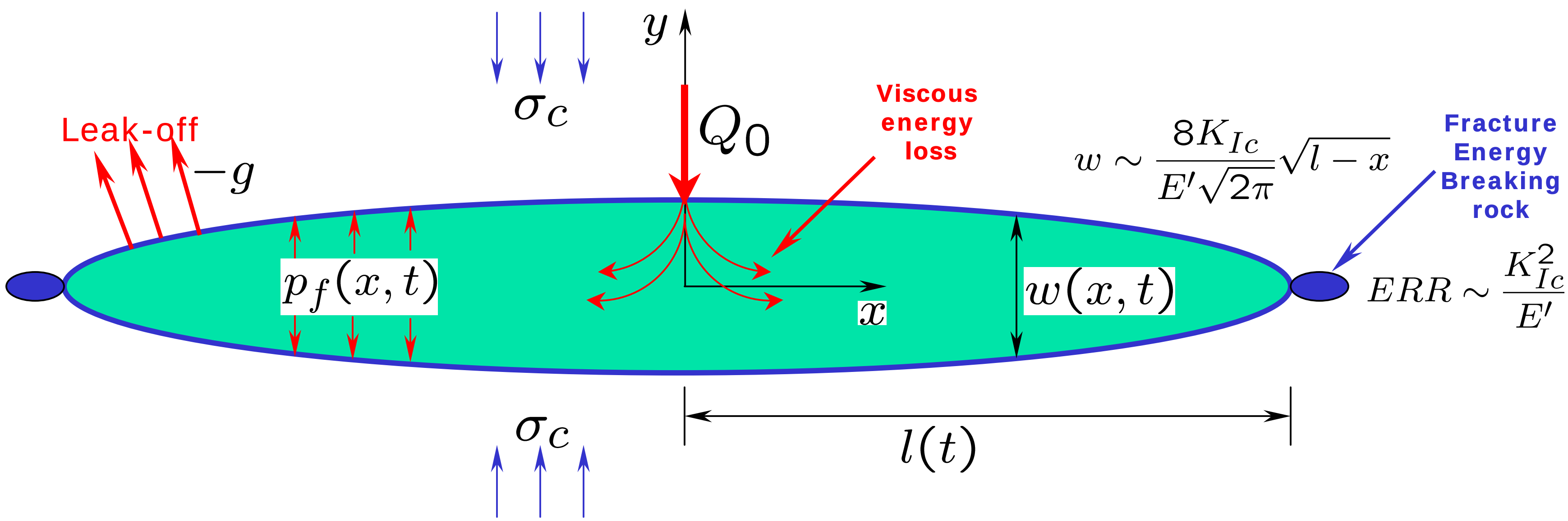
An actual hydraulic fracture

HF experiment (Jeffrey *et al* CSIRO)



1D model and physical processes

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\left(\frac{w^3}{\mu'}\frac{\partial p}{\partial x}\right) + Q_0\delta(x) - g; \quad w^3\frac{\partial p}{\partial x}\Big]_{\pm l} = 0$$



$$p_f(x,t) - \sigma_c = -\frac{E'}{4\pi}\int_{-l}^{l}\frac{\partial w}{\partial \xi}\frac{d\xi}{\xi - x}; \quad w(\pm l, t) = 0$$

Scaling and dimensionless quantities

- Rescale: $\xi = x/l, \quad l = L\gamma, \quad w = \epsilon L \Omega, \quad p = \epsilon E' \Pi$
- Dimensionless quantities:

$$\mathcal{G}_v = \frac{Q_0 t}{\epsilon L^2}, \; \mathcal{G}_m = \frac{\mu'}{\epsilon^3 E' t}, \; \mathcal{G}_k = \frac{K'}{\epsilon E' L^{1/2}}, \; \mathcal{G}_c = \frac{C' t^{1/2}}{\epsilon L}$$

- Governing equations become:

$$\frac{t(\epsilon L)_t}{\epsilon L}\Omega + \dot{\Omega} t - \frac{t(L\gamma)_t}{L\gamma}\xi\frac{\partial \Omega}{\partial \xi} + \frac{\mathcal{G}_c}{\sqrt{1 - \frac{t_0(\xi l)}{t}}} = \frac{1}{\mathcal{G}_m \gamma^2}\frac{\partial}{\partial \xi}\left(\Omega^3 \frac{\partial \Pi}{\partial \xi}\right)$$

$$\Pi = -\frac{1}{2\pi\gamma}\int_0^1 \frac{\partial \Omega}{\partial \chi}\frac{d\chi}{\chi - \xi} \qquad\qquad \Omega \overset{\xi \to \pm 1}{=} \mathcal{G}_k \gamma^{1/2}\sqrt{1 - \xi}$$

$$\mathcal{G}_v = 2\gamma \int_0^1 \Omega d\chi + 2\mathcal{G}_c \frac{1}{L}\int_0^1 l(\theta t)\theta^{-1/2}\int_0^1 \Gamma d\chi d\theta$$

Two of the physical processes

- Large toughness: $\mathcal{G}_k = 1;\ \mathcal{G}_m \ll 1;\ \mathcal{G}_c \ll 1$

$$\frac{1}{\gamma^2}\frac{\partial}{\partial \xi}\left(\Omega^3 \frac{\partial \Pi}{\partial \xi}\right) \approx 0 \ \Rightarrow\ \Pi \sim c_0; \quad \Omega \sim c_1\sqrt{1-\xi}$$

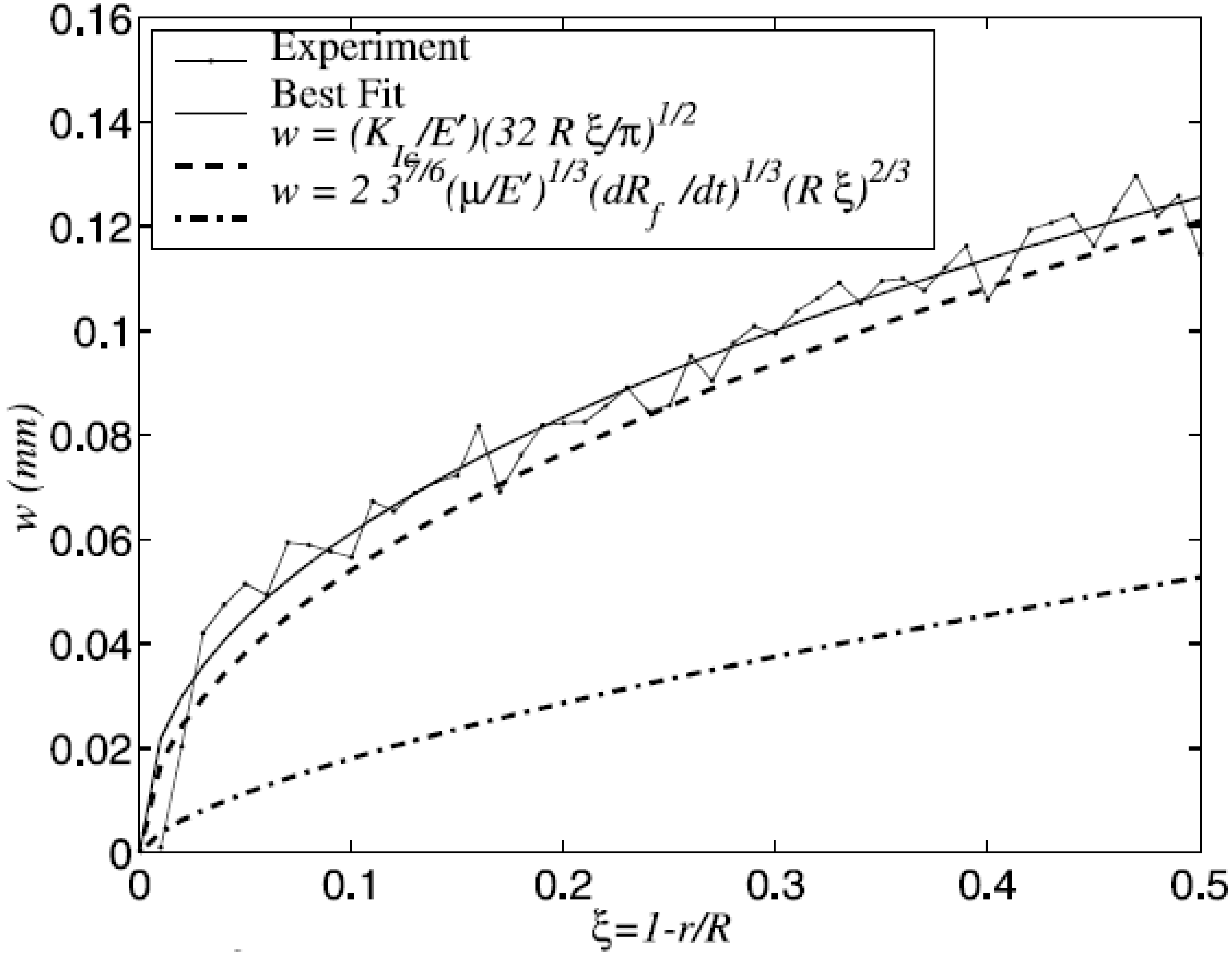
- Large viscosity: $\mathcal{G}_k \ll 1;\ \mathcal{G}_m = 1;\ \mathcal{G}_c \ll 1$

$$\frac{t(L\gamma)_t}{L\gamma}\xi\frac{\partial \Omega}{\partial \xi} \approx \frac{1}{\gamma^2 \mathcal{G}_m}\frac{\partial}{\partial \xi}\left(\Omega^3 \frac{\partial \Pi}{\partial \xi}\right)$$

$$\overset{\xi \to 1}{\Rightarrow} \qquad \Pi \sim c_2 (1-\xi)^{-1/3}$$

$$\Omega \sim c_3 (1-\xi)^{2/3}$$

Experiment I (Bunger *et al* 2004)



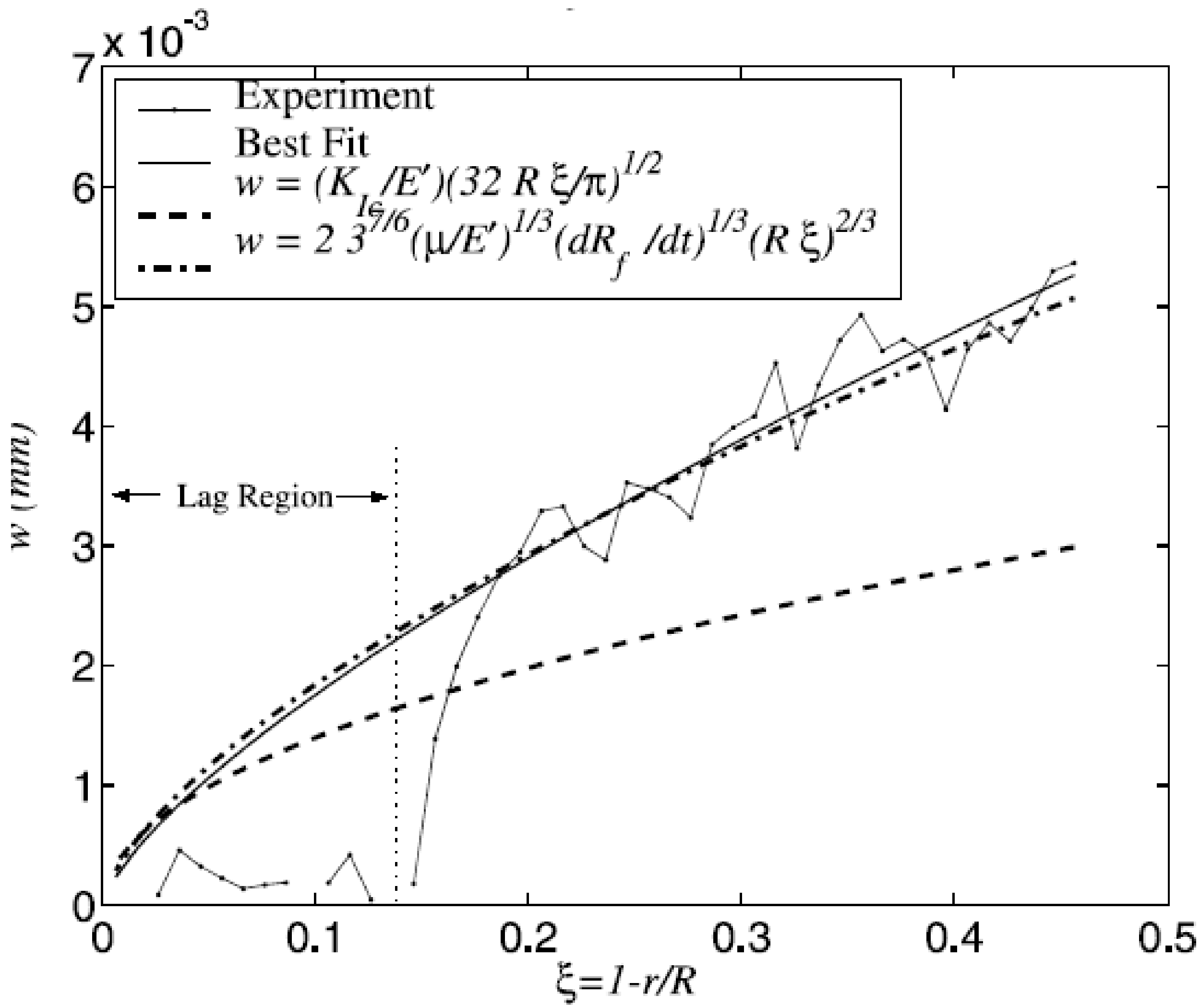
Theory

Large toughness:

(Spence & Sharp)

$$\Omega \approx c_1\sqrt{1-\xi}$$

Experiment II (Bunger *et al* 2004)



Theory
Large viscosity:
(Desroches *et al*)

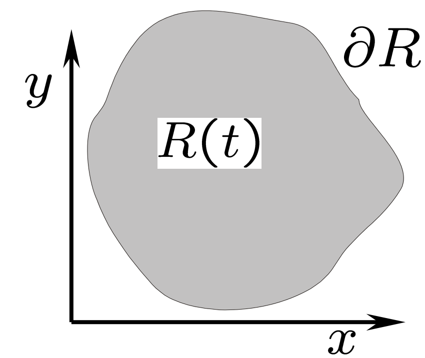
$$\Omega \sim c_3 (1-\xi)^{2/3}$$

The coupled EHF equations in 2D

- The elasticity equation – balance of forces

$$\int_{R(t)} C(x,y;\xi,\eta)w(\xi,\eta,t)d\xi d\eta = p_f(x,y,t) - \sigma_c$$

$$\Rightarrow \quad Cw = p_f - \sigma_c = p$$



- The fluid flow equation – mass balance

$$\frac{\partial w}{\partial t} = \nabla \cdot \left(\frac{w^3}{\mu'} \nabla p\right) + \delta(x,y)Q_0 - g$$

$$\Rightarrow \quad \frac{\Delta w}{\Delta t} = A(w)p + S$$

Boundary & propagation conditions

- Boundary conditions

$$w|_{\partial R} = 0; \qquad w^3 \frac{\partial p}{\partial n}|_{\partial R} = 0$$

- Propagation condition

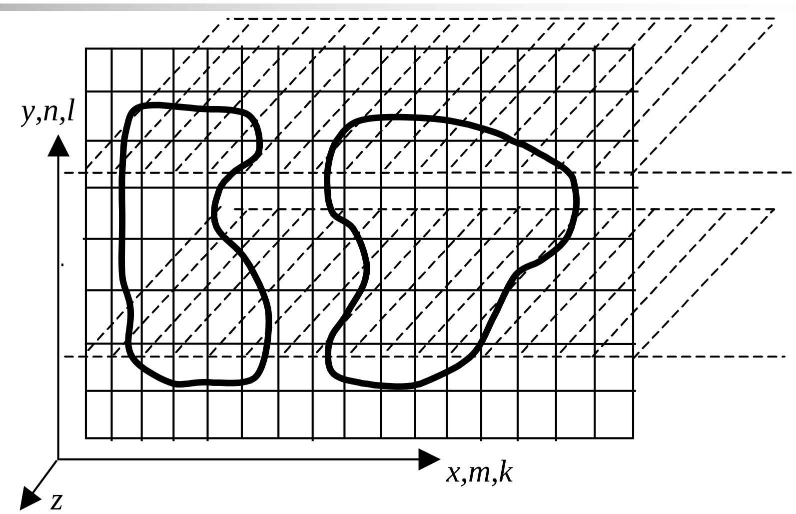
$$\int_{R(t)} K(x,y;\xi,\eta)p(\xi,\eta,t)d\xi d\eta = K_I(x,y,t) = K_{Ic}(x,y)$$

The elasticity equation $\quad Cw = p$

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2G\epsilon_{ij}$$

$$\sigma_{ij,j} + f_i = 0$$

$$\Downarrow$$



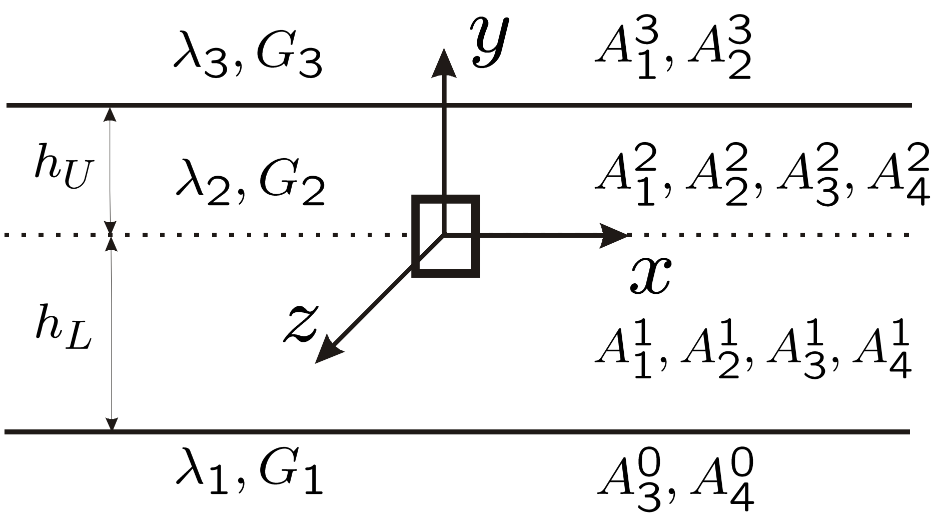
$$\partial_y T = \mathcal{K} T + b \overset{FT}{\Rightarrow} \partial_y \widehat{T} = \widehat{\mathcal{K}}(k)\widehat{T} + \widehat{b}$$

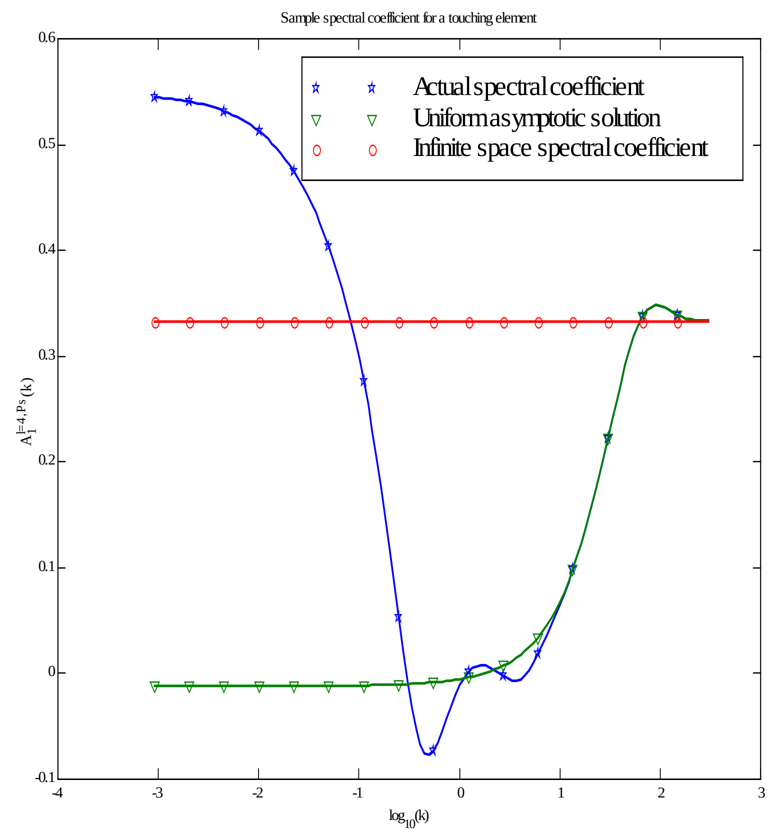
$$\begin{bmatrix} T_s(k) \\ T_t(k) \end{bmatrix} = \begin{bmatrix} Z_s & 0 \\ 0 & Z_t \end{bmatrix} \begin{bmatrix} A_s(k) \\ A_t(k) \end{bmatrix}$$

$$Z_{ij} = (c_1 + c_2 k y)\, e^{\pm k y}$$

3 Layer Uniform Asymptotic Solution

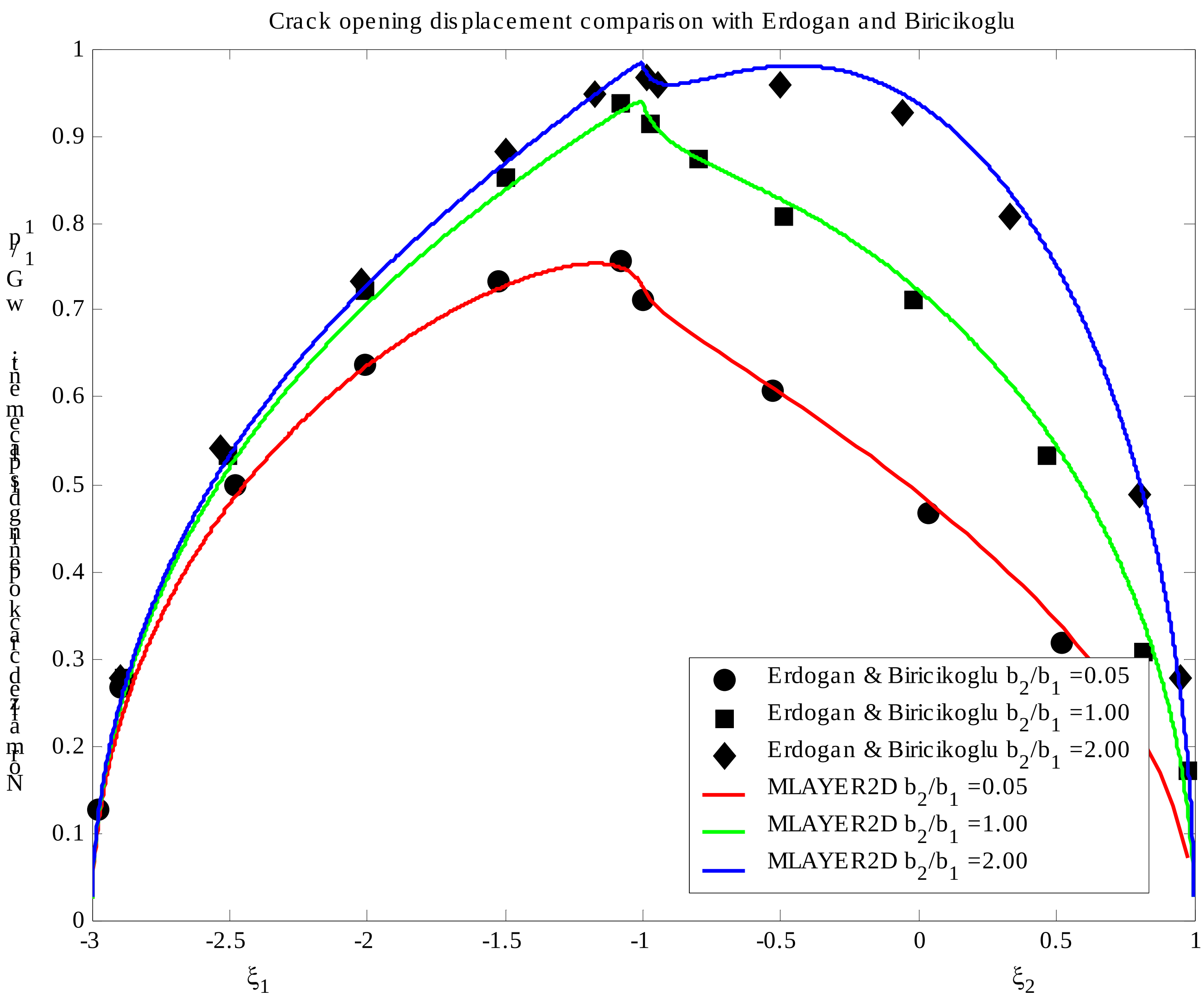
$$A_j^{l,\mu}(k) \overset{k\to\infty}{\sim} A_j^{l,U}(k) + A_j^{l,L}(k) - A_j^{l,\infty}(k)$$

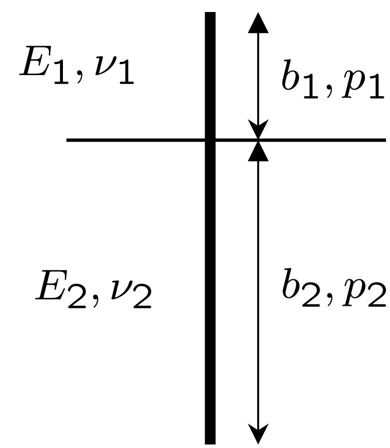




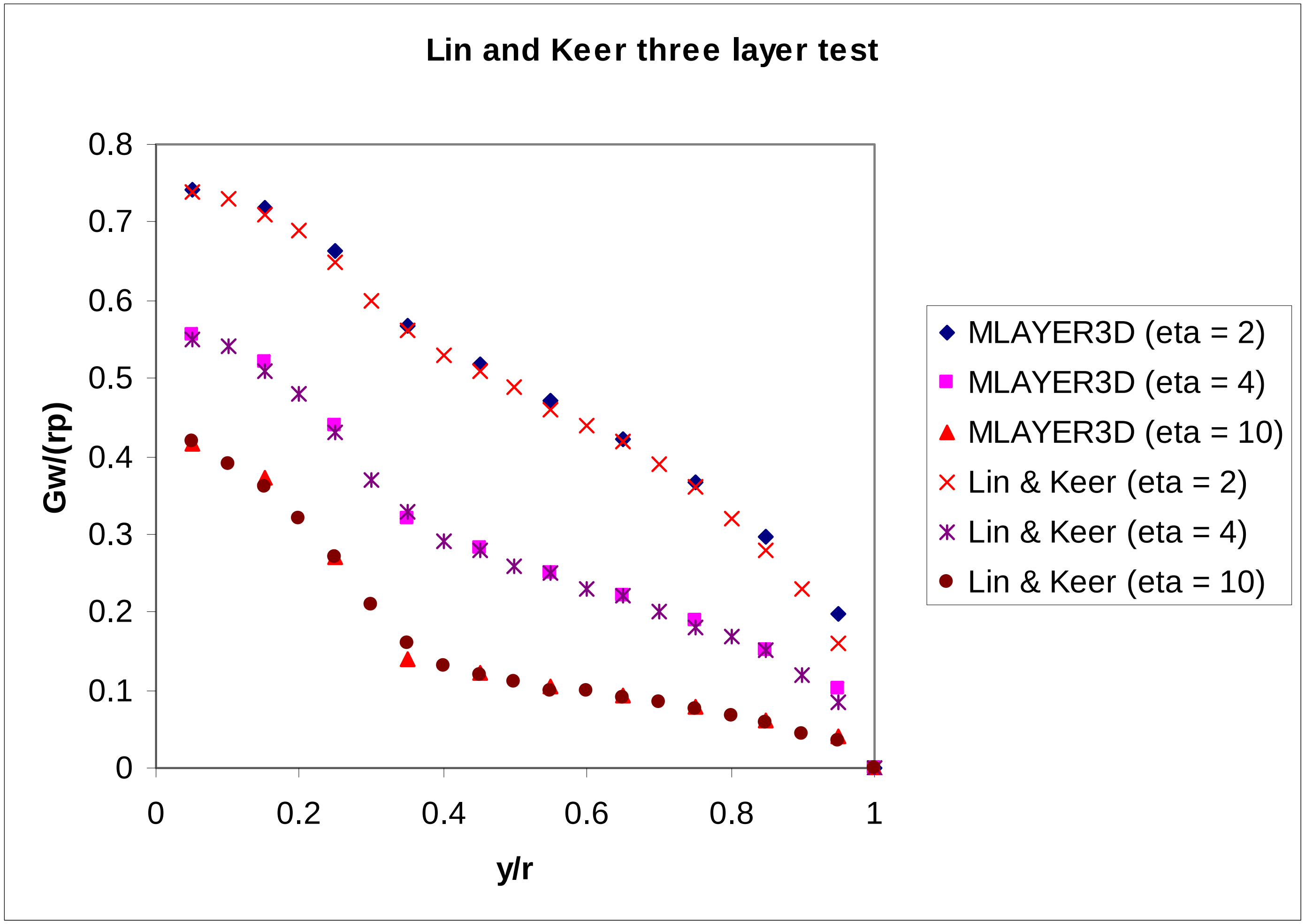
$$k\widehat{\sigma}_{zz} = fn^2 \left\{ A_1^{Ps}(k) \right\} e^{-ky} + \ldots$$

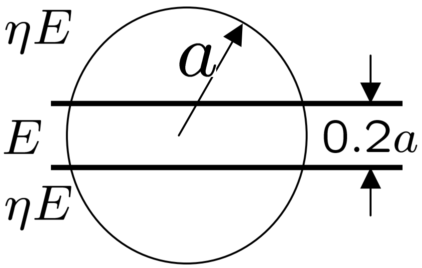
Crack cutting an interface





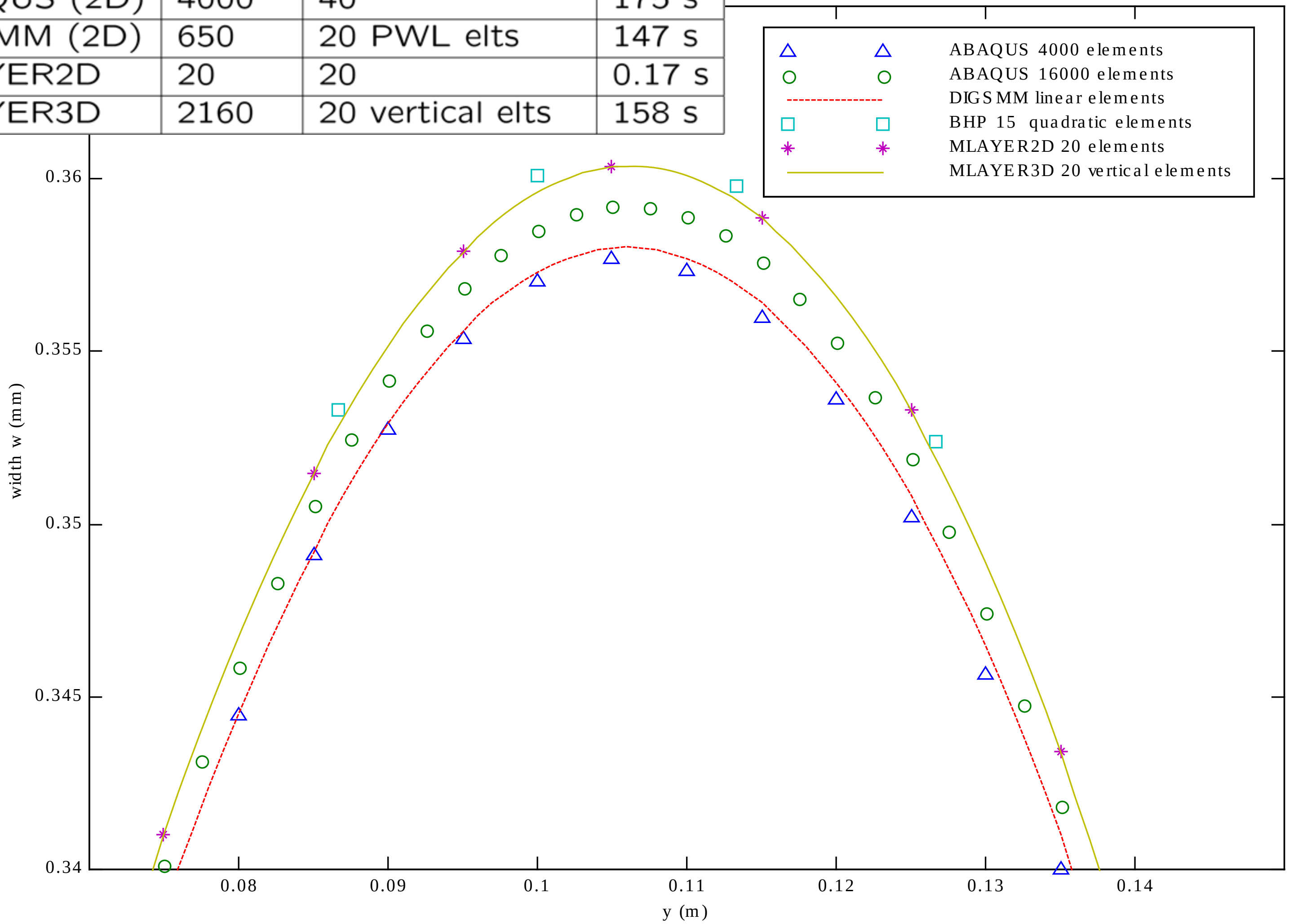
A penny crack cutting 2 interfaces

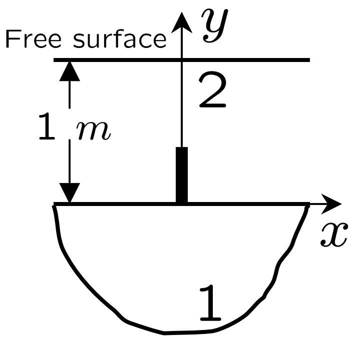




A crack touching an interface

Algorithm	# elts.	# elts. on crack	Time
ABAQUS (2D)	4000	40	175 s
DIGSMM (2D)	650	20 PWL elts	147 s
MLAYER2D	20	20	0.17 s
MLAYER3D	2160	20 vertical elts	158 s

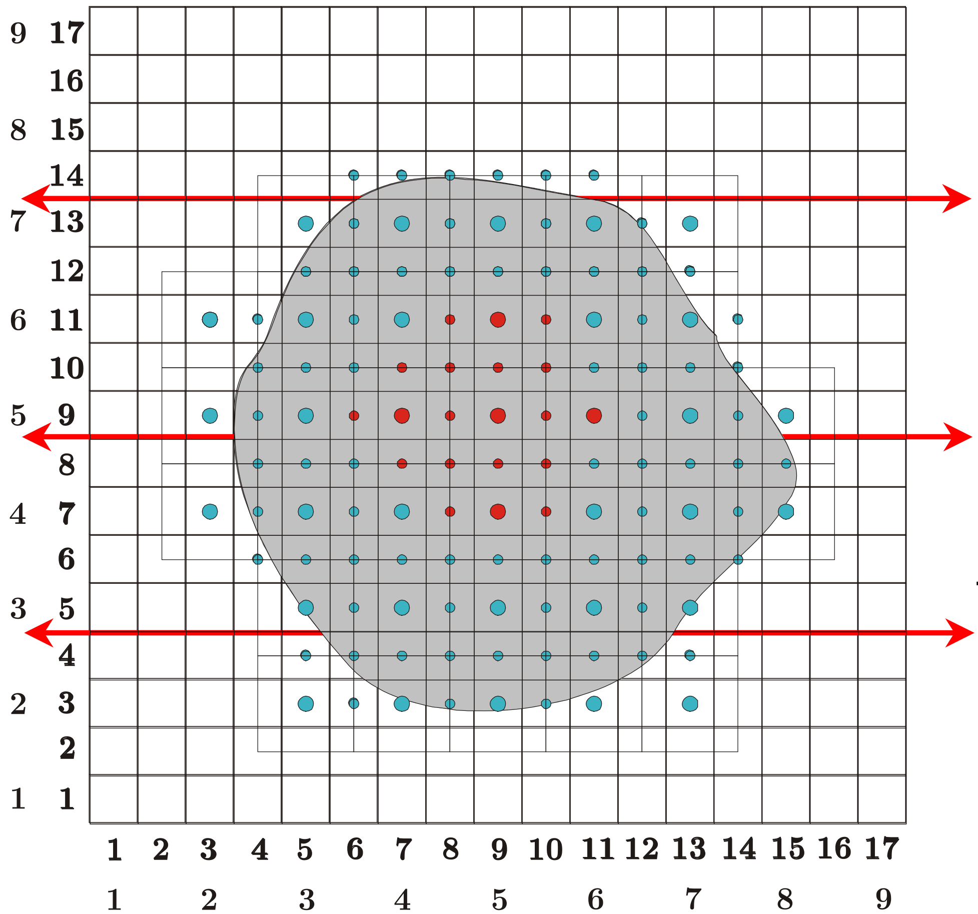




$E_1 = 2000 MPa$
$\nu_1 = 0.1$

$E_2 = 1000 MPa$
$\nu_2 = 0.3$
$l = 0.2\ m$
$\sigma_n = 1\ MPa$
$\sigma_s = 0\ MPa$

Eulerian approach & Coupled HF Equations



$$Cw = p; \quad w \geq w_c$$

$$+$$

$$\frac{\partial w}{\partial t} = A(w)p + S$$

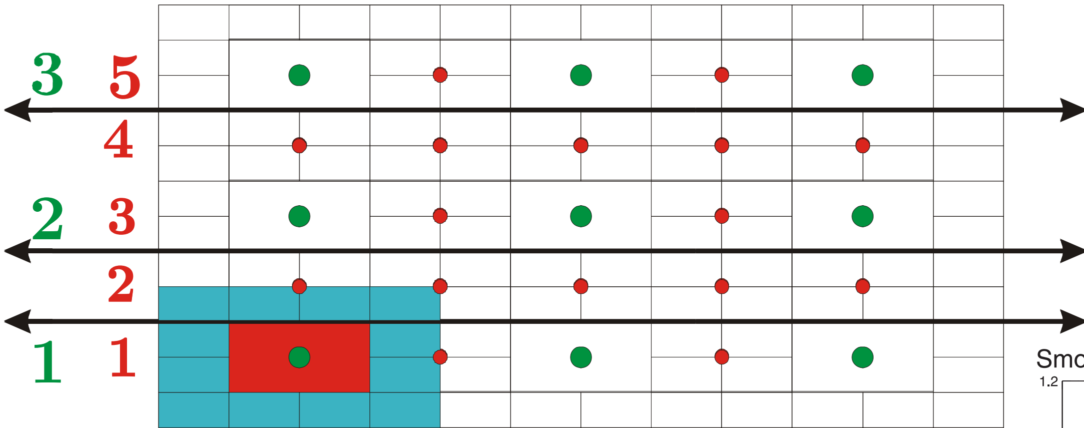
$$\Downarrow$$

$$\frac{\partial w}{\partial t} = A(w)Cw + S$$

$$\Delta t < \frac{\Delta x^3}{E'\bar{D}}$$

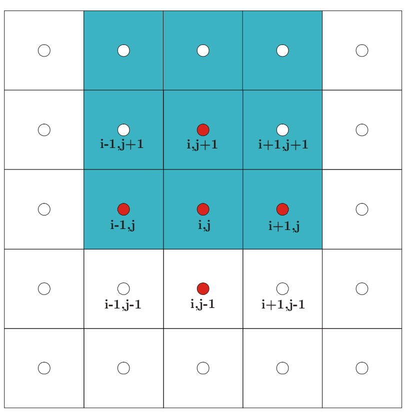
MG preconditioning of $\frac{\Delta w}{\Delta t} = A(w)Cw + S$

- C coarsening using dual mesh

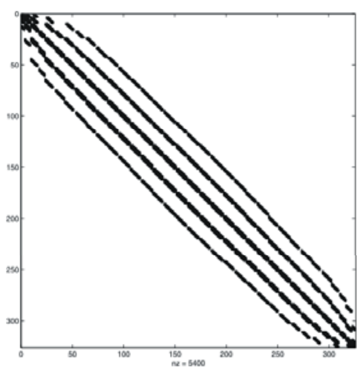


$$C_{ij}^{2h} = C_{ij}^{h} + \sum_{k \in N_j} C_{ik}^{\frac{h}{2}}$$

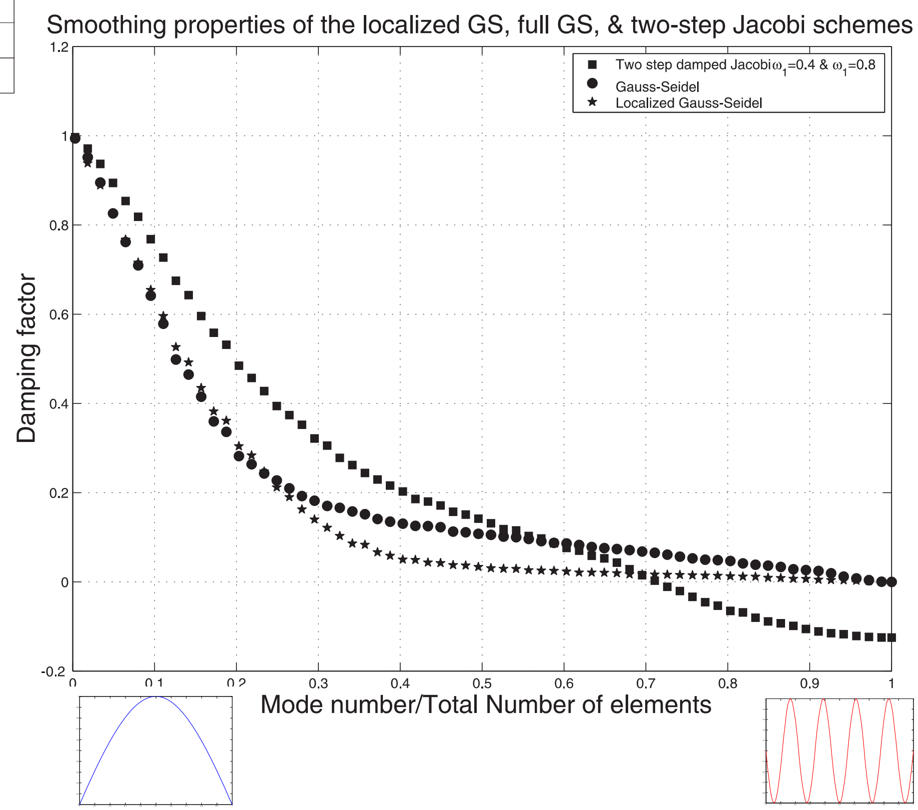
- Localized GS smoother



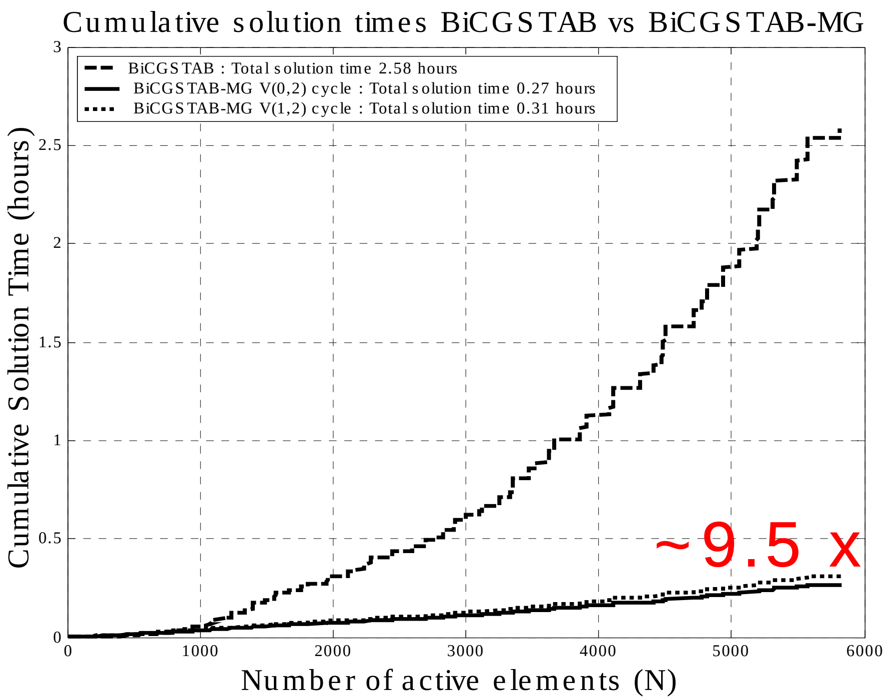
$\Rightarrow$

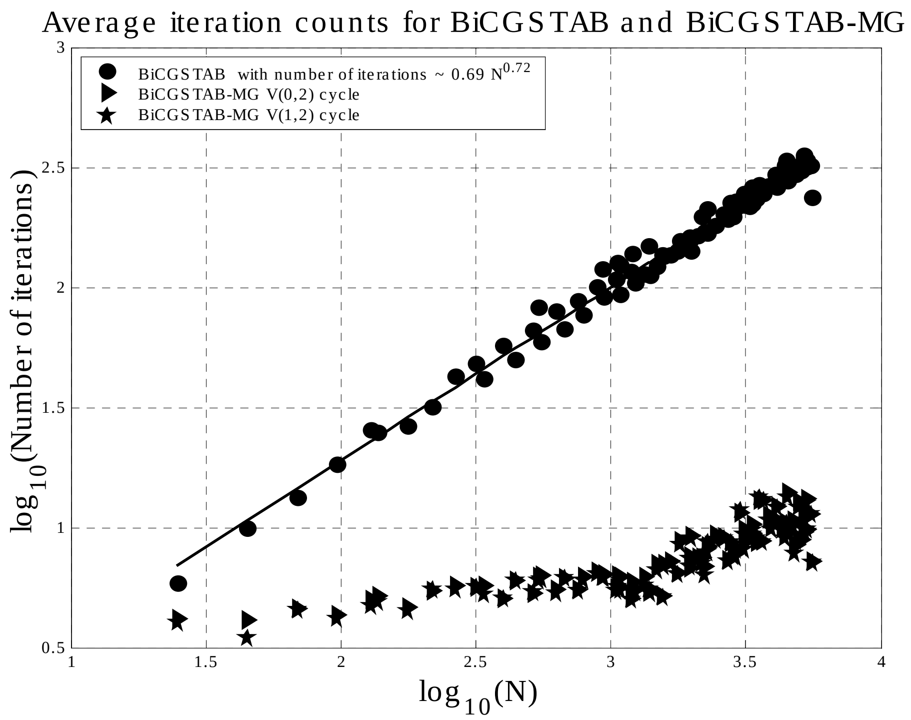


$$[AC]_{MN} \approx \sum_{K \in \mathcal{N}_M^5} A_{MK} C_{KN} \text{ when } N \in \mathcal{N}_K^9$$

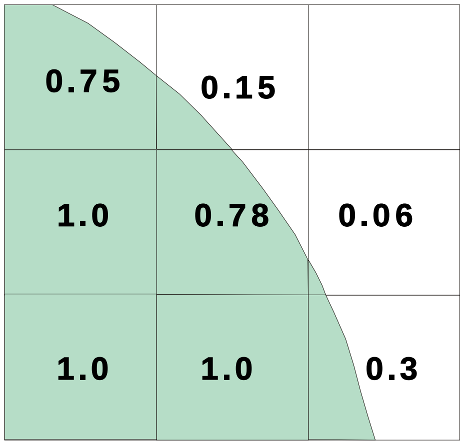


Performance of MG Preconditioner





Front evolution via the VOF method



$$\frac{\partial \chi}{\partial t} + \mathbf{v} \cdot \nabla \chi = 0$$

$$\nabla \cdot \mathbf{v} = 0$$

$$A \frac{\partial}{\partial t} \left(\frac{1}{A} \int_A \chi \, dA \right) = - \int_A \nabla \cdot (\chi \mathbf{v}) \, dA$$

$$\frac{\partial F}{\partial t} = -\frac{1}{A} \int_{\partial A} \chi v_n dl$$

Time stepping and front evolution

Time step loop: $t \leftarrow t + \Delta t$

VOF loop:

Coupled Solution

$$\left.\begin{aligned} \frac{\Delta w}{\Delta t} &= A(w)Cw + S \\ p &= Cw \end{aligned}\right] v = -\frac{w^2}{\mu'}\nabla p$$

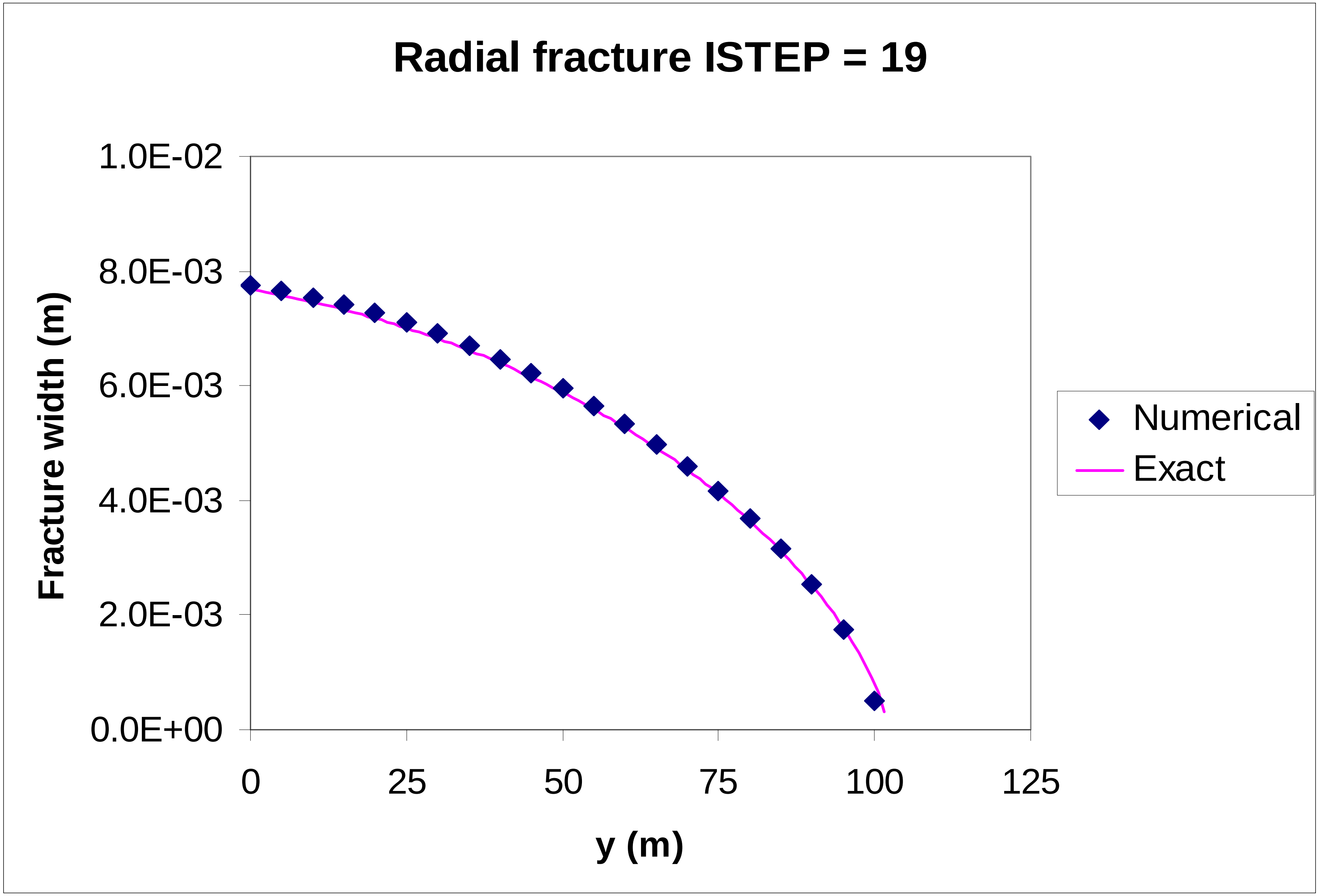
end

$$F_{k+1} = F_k - \frac{\Delta t}{A}\int_{\partial A} \chi v_n dl$$

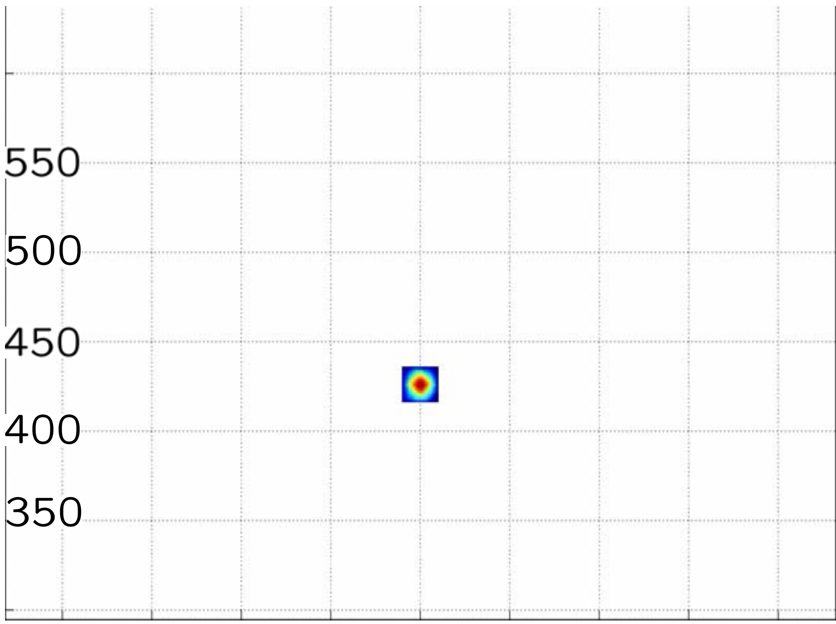
next VOF iteration

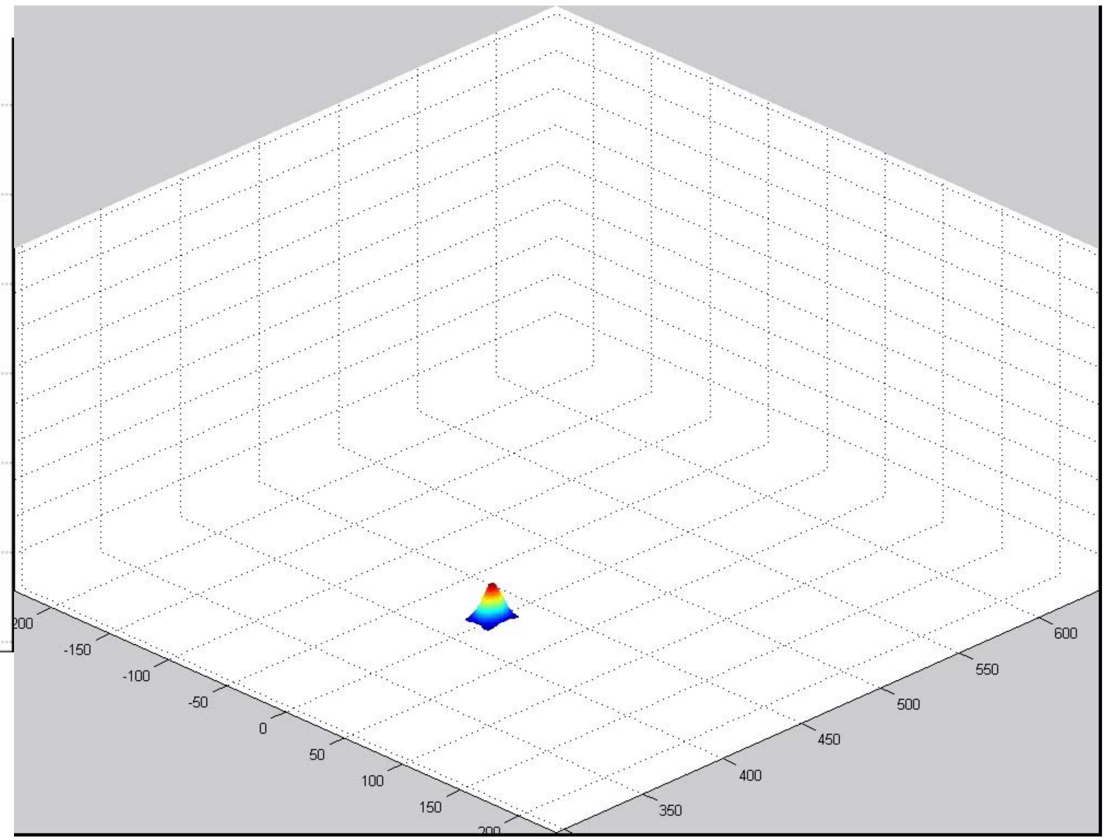
next time step

Radial Solution

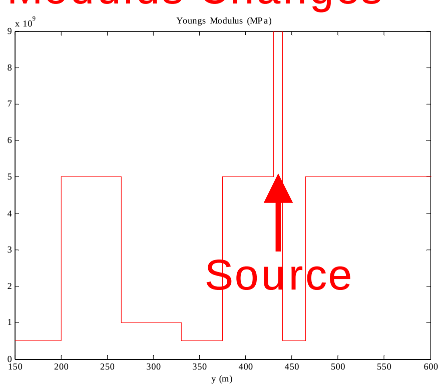


Fracture width for modulus contrast

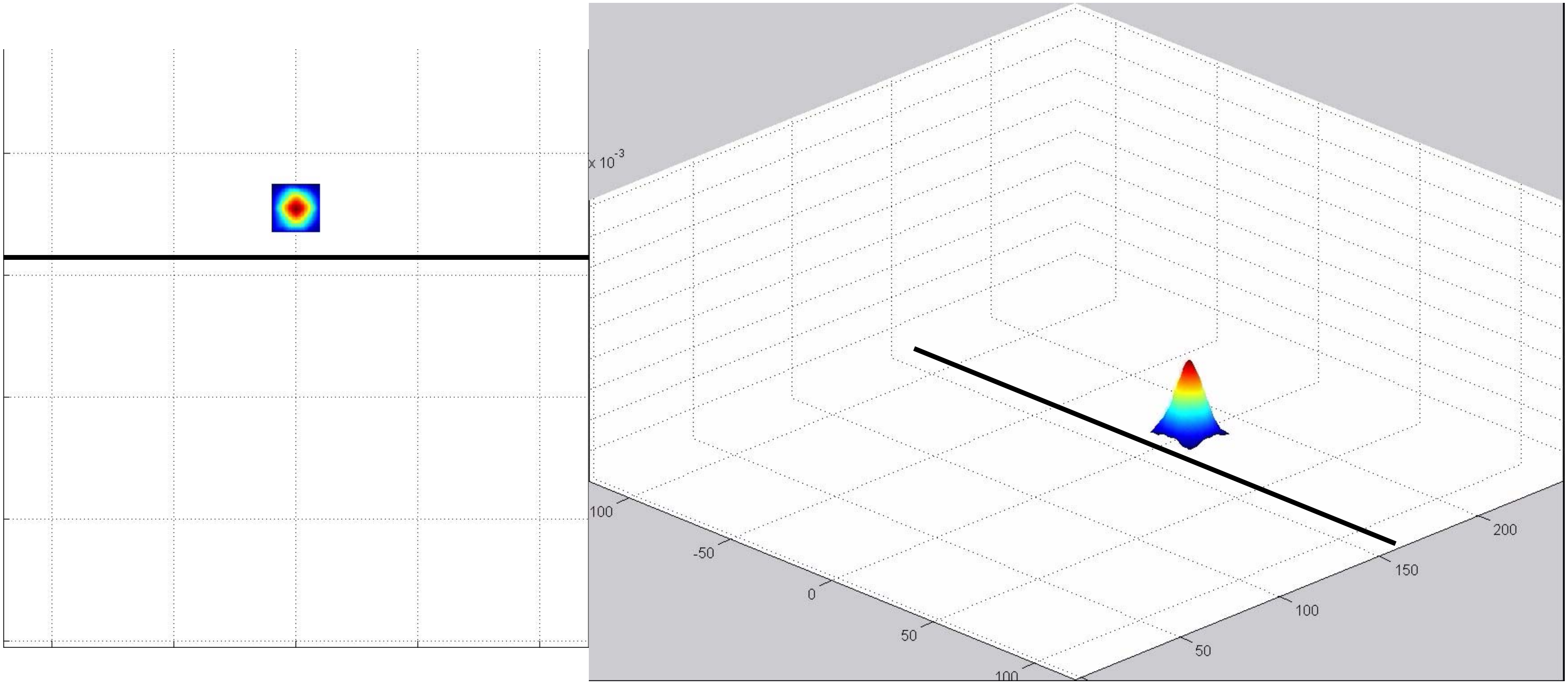




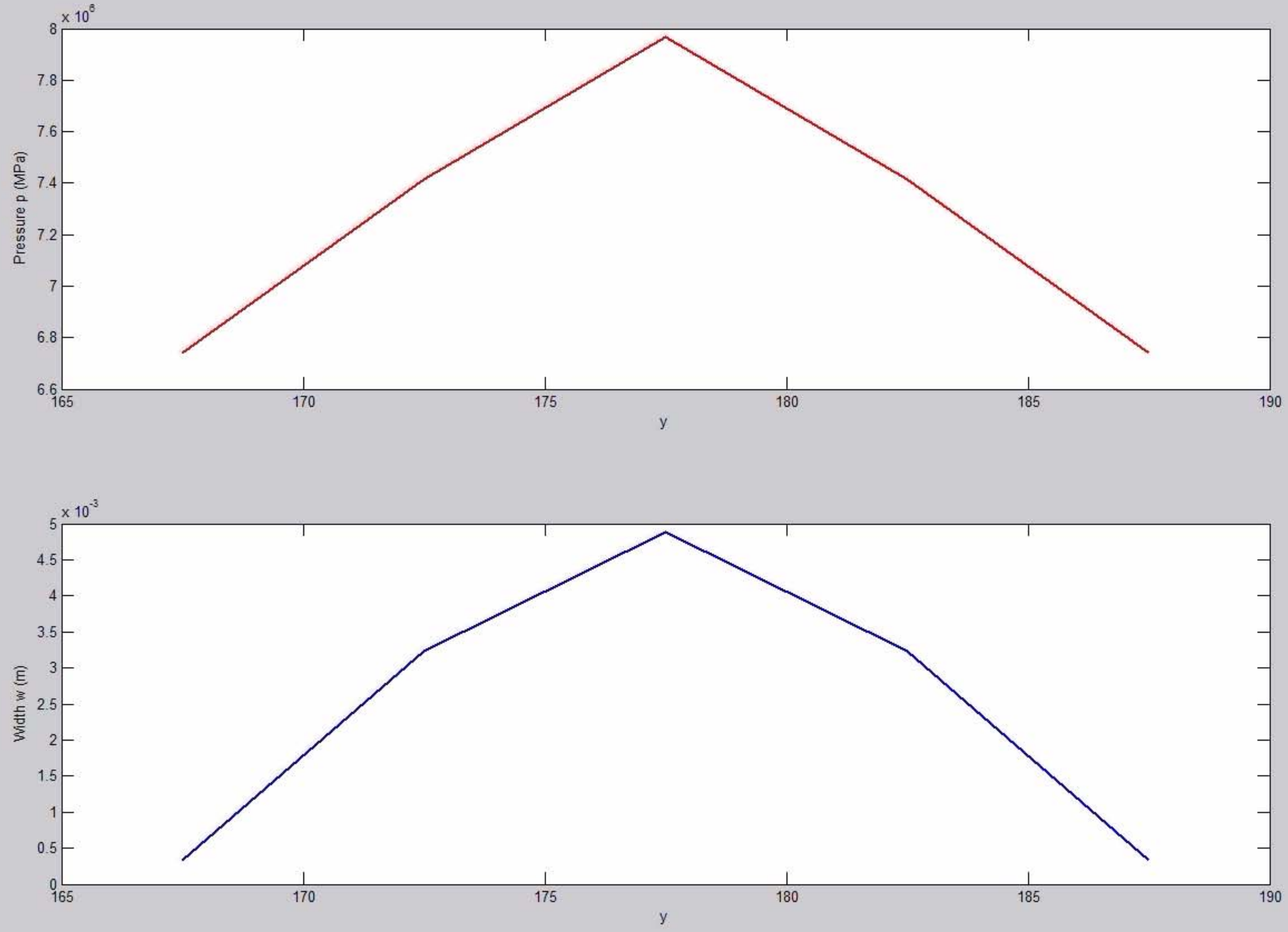
Modulus Changes



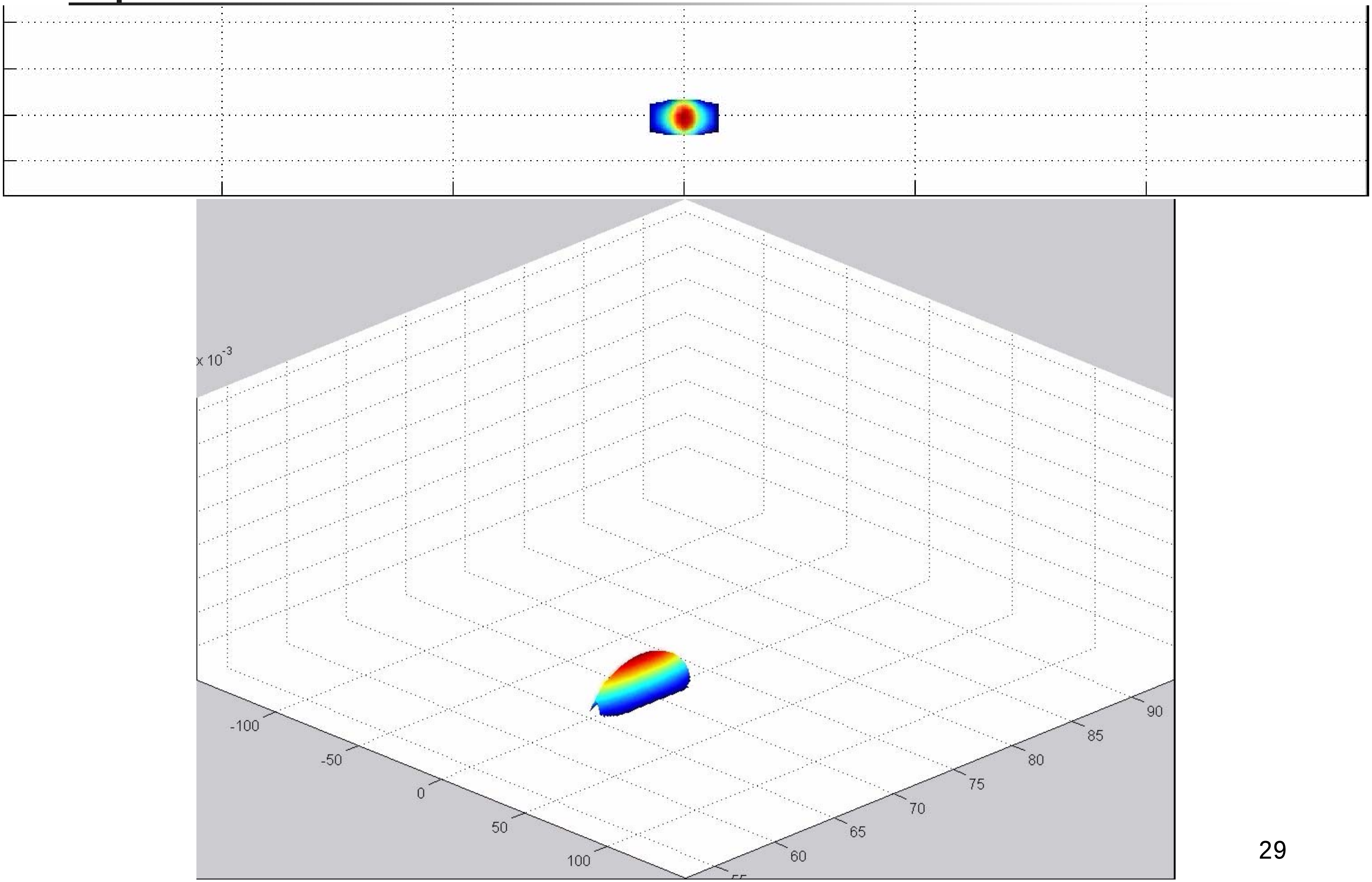
Fracture width for stress jump



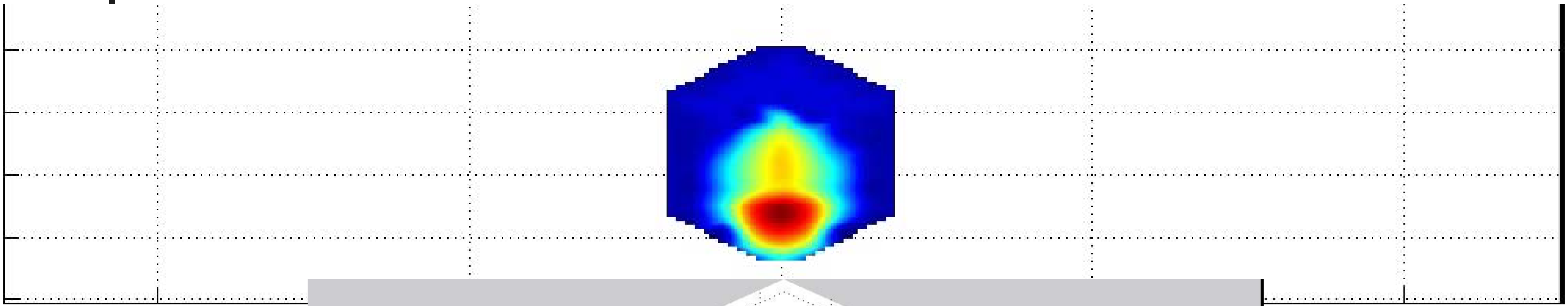
Pressure and width evolution

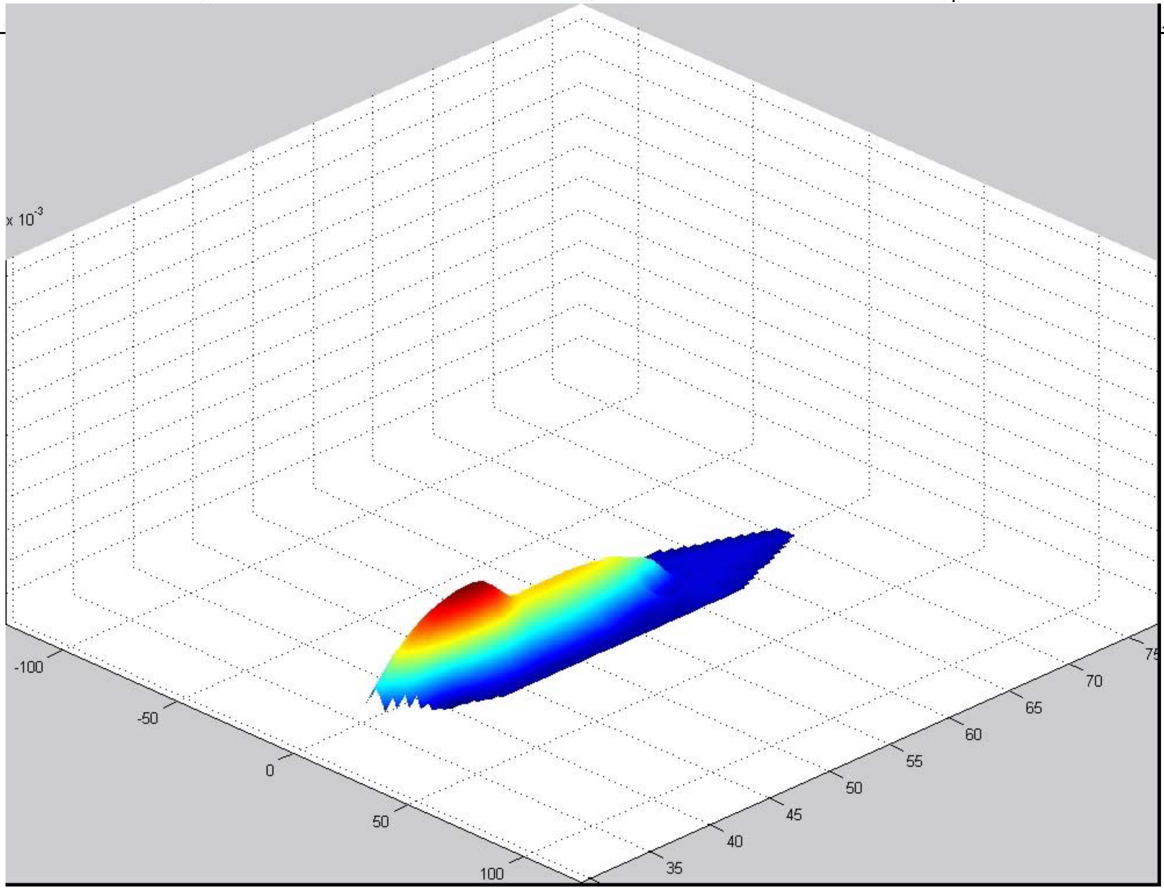


Channel fracture and breakout



Hourglass HF with leakoff





Concluding remarks

- Examples of hydraulic fractures
- Scaling and physical processes in 1D models
  - Tip asymptotics: $\Omega \approx c_1 \sqrt{1-\xi}$ & $\Omega \sim c_3 (1-\xi)^{2/3}$
  - Experimental verification
- Numerical models of 2-3D hydraulic fractures
  - The non-local elasticity equation
  - An Eulerian approach and the coupled equations
  - FT construction of the elasticity influence matrices
  - A multigrid algorithm for the coupled problem
  - Front evolution via the VOF method
- Numerical results