Math 257/316 Assignment 8
Not to be handed in

Problem 1: Consider an infinite string subject to the initial condition

$$u(x,0) = \begin{cases} x+1 & \text{if } -1 < x < 0 \\ 1-x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \qquad\qquad u_t(x,0) = 0.$$

Sketch the shape of the string for $t = 0$, $t = 1/2a$, $t = 1/a$ and $t = 2/a$.

Problem 2: Suppose an infinite string is hit with a hammer, so that the initial conditions are given by

$$u(x,0) = 0, \qquad u_t(x,0) = \begin{cases} 1 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the shape of the string for all later times.

Problem 3: Consider an elastic string of length 1 whose ends are held fixed. The string is set in motion with initial velocity $g(x)$ from an initial position $f(x)$. For each of the following functions, find the corresponding displacement $u(x,t)$ of the string for all time.

(a) $f(x) = 0 \qquad g(x) = \begin{cases} 0, & 0 \le x \le 1/2, \\ 1, & 1/2 < x < 1. \end{cases}$

(b) $f(x) = 2\sin(\pi x) + \sin(3\pi x), \qquad g(x) = \sin(3\pi x)$

Problem 4: If an elastic string is free at one end, the boundary condition to be satisfied there is that $u_x = 0$.

(a) Write the Initial/Boundary Value Problem for the displacement $u(x,t)$ of an elastic string of length L, fixed at $x = 0$ and free at $x = L$, set in motion with no initial velocity from the initial position $u(x,0) = f(x)$

(b) Find the General Solution for this problem, using separation of variables.

(c) Find the displacement $u(x,t)$ of the elastic string if the initial position is given by

$$u(x,0) = f(x) = 2\sin\left(\frac{9\pi}{2L}x\right)$$

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Problem 5: (Do not hand in) Solve the following inhomogeneous initial boundary value problem for the wave equation:

$$
\begin{aligned}
u_{tt} &= c^2 u_{xx} + e^{-t}\sin(5x)\,, \ 0 < x < \frac{\pi}{2},\ t > 0 \\
u(0,t) &= 0 \text{ and } u_x(\frac{\pi}{2}, t) = t, \quad t > 0 \\
u(x,0) &= 0, \qquad u_t(x,0) = \sin 3x + x, \quad 0 < x < \frac{\pi}{2}
\end{aligned}
$$