Math 257-316 Assignment 4 - Supplementary Problems

- This a supplementary assignment and must not be handed in.
- The first midterm exam is on Mon., Oct. 22 th - practice exams on web site.

1. [**Separation of variables**]
2. Determine whether the method of separation of variables can be used to replace the following PDE's by a pair of ODE's. If so, find the equations.

   (a) $xu_{xx} + tu_t = 0$. (b) $u_{xx} + u_{yy} = x$. (c) $u_x + u_{xt} + u_t = 0$.
3. Find all eigenvalues and corresponding eigenfunctions for the following problem
$$-y'' = \lambda y \quad (0 < x < 1), \quad y(0) = 0, \quad y'(1) = 0.$$
4. For each (real) constant k find all the non-zero solutions of the following boundary value problem
$$X'' = kX, \text{ for } x \in (0,1), \qquad X(0) = -X(1).$$
5. Apply the method of separation of variables to determine a solution to the one dimensional heat equation with homogeneous Mixed boundary conditions, i.e.
$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$
$$\text{BC: } \frac{\partial u(0,t)}{\partial x} = 0 = u(L,t)$$
$$\text{IC: } u(x,0) = f(x)$$

   Now evaluate the coefficients of the series for the initial condition function $f(x) = x$. Assume $t = 0$ and $L = 1$ and plot the sum of the first 5 terms of the series over the interval $[-2, 2]$.
6. [**Periodic functions**]
7. Which of the following functions are periodic? For those that are, find the fundamental period: (a) $\sin x \cos x$ (b) $\sec x + \tan \sqrt{2}x$ (c) $\sin(x^2)$.
8. [**Fourier series**]
9. Find the Full Fourier series expansion of the function $f(x) = x^2$ for $-L < x < L$ and which has a period of $2L$. Use this series to determine a series expansion for $\frac{\pi^2}{6}$.

   (a) Show that $\cos x = \dfrac{8}{\pi} \displaystyle\sum_{n=1}^{\infty} \frac{n}{(2n)^2 - 1} \sin(2nx), \; 0 < x < \pi$.

   (b) The series in (a) converges for all x. What function does it converge to?

   (c) Show that $\displaystyle\sum_{n=1,3,5,\ldots} \frac{n}{4n^2 - 1} (-1)^{(n-1)/2} = \frac{\pi}{8.2^{1/2}}$.