Math 257/316, Midterm 1, Section 101
9 am on 21st October 2013

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 50.

1. Consider the second order differential equation:

$$Ly = 3xy'' + y' - y = 0 \tag{1}$$

   (a) Classify the points $0 \leq x < \infty$ as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation.

   [7 marks]

   (b) If you were given $y(-2) = 1$ and $y'(-2) = 0$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

   [3 marks]

   (c) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

   [20 marks]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following mixed boundary conditions:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi,\ t > 0 \\ \text{BC} &: \quad u(0,t) = 0 = \frac{\partial u(\pi,t)}{\partial x} \\ \text{IC} &: \quad u(x,0) = x \end{aligned}$$

   Hint: It may be useful to know that

$$\int_0^\pi x \sin\left(\frac{(2n+1)x}{2}\right) dx = \frac{4(-1)^n}{(2n+1)^2}$$

   [20 marks]