Math 257/316, Midterm 1, Section 101
9 am on 18 th of October 2017

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 50.

1. Consider the second order differential equation:

$$Ly = 2x^2y'' + 3xy' + (2x-1)y = 0 \tag{1}$$

   (a) Classify the points $0 \leq x < \infty$ as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation.

   [7 marks]

   (b) If you were given $y(-1) = 1$ and $y'(-1) = 0$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

   [3 marks]

   (c) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case. What is the minimal radius of convergence of these series?

   [20 marks]

2. Apply the method of separation of variables to determine the temperature $u(x,t)$ in a rod of length $\pi/2$ that involves a chemical reaction that generates heat at a rate proportional to the temperature, maintained at a zero temperature at the left endpoint, and insulated at the right endpoint. The initial-boundary value problem satisfied by $u(x,t)$ is given by:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \gamma^2 u, \quad 0 < x < \pi/2, \ t > 0 \\ \text{BC} &: \ u(0,t) = 0 \text{ and } 0 = \frac{\partial u(\pi/2,t)}{\partial x} \\ \text{IC} &: \ u(x,0) = \sin 3x \end{aligned}$$

   Please show all the cases when solving the appropriate eigenvalue problem.

   Hint: When separating the variables, group the γ^2 term with the time ordinary differential equation.

   [20 marks]