Math 257/316, Midterm 1, Section 102
22 October 2012

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 50.

1. Consider the second order differential equation:

$$Ly = 3x^2y'' - (1-x)xy' + y = 0 \tag{1}$$

   (a) Classify the points $0 \le x < \infty$ as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation.

   [7 marks]

   (b) If you were given $y(1) = 3$ and $y'(1) = -1$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

   [3 marks]

   (c) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

   [20 marks]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following Neumann boundary conditions:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \ t > 0 \\ \text{BC} &: \frac{\partial u(0,t)}{\partial x} = 0 = \frac{\partial u(\pi,t)}{\partial x} \\ \text{IC} &: u(x,0) = x \end{aligned}$$

[20 marks]