

Math 257/316, Midterm 1, Section 102
4 pm on 8 th October 2014

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 50.

1. Consider the second order differential equation:

$$Ly = 6x^2y'' + 7xy' + (x - 1)y = 0 \quad (1)$$

- (a) Classify the points $0 \leq x < \infty$ as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation. [7 marks]
- (b) If you were given $y(1) = 1$ and $y'(1) = 0$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series? [3 marks]
- (c) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

[20 marks]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following periodic boundary conditions:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, & -\pi < x < \pi, t > 0 \\ \text{BC} &: u(-\pi, t) = u(\pi, t) \text{ and } \frac{\partial u(-\pi, t)}{\partial x} = \frac{\partial u(\pi, t)}{\partial x} \\ \text{IC} &: u(x, 0) = x \end{aligned}$$

[20 marks]