

Math 257/316, Midterm 1, Section 102
4 pm on 21 st October 2015

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 50.

1. Consider the second order differential equation:

$$Ly = 8x^2y'' + 10xy' - (1 - x)y = 0 \tag{1}$$

- (a) Classify the points $0 \leq x < \infty$ as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation. [7 marks]
- (b) If you were given $y(1) = 1$ and $y'(1) = 0$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series? [3 marks]
- (c) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case. What is the minimal radius of convergence of these series?

[20 marks]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with heat loss to the surrounding medium at a rate $\gamma > 0$ subject to the following boundary conditions:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} - \gamma u, & 0 < x < \pi/2, t > 0 \\ \text{BC} &: \frac{\partial u(0, t)}{\partial x} = 0 \text{ and } 0 = u(\pi/2, t) \\ \text{IC} &: u(x, 0) = \cos 3x \end{aligned}$$

Please show all the cases when solving the appropriate eigenvalue problem.

Hint: When separating the variables group the γ term with the time ordinary differential equation.

[20 marks]