Math 257/316, Midterm 1, Section 201
15 February 2012

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 80.

1. Consider the second order differential equation:

$$Ly = 16x^2(1+x)y'' + 16xy' - y = 0 \tag{1}$$

   (a) Classify the points $0 \le x < \infty$ as ordinary points, regular singular points, or irregular singular points.

   [10 marks]

   (b) If you were given $y(1) = 1$ and $y'(1) = 0$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

   [5 marks]

   (c) If you required a solution about $x = -1$, what form of series expansion would you assume (**Do not** determine the expansion coefficients of this series)? What is the behavior of the solution as $x \to -1$?What would be the minimal radius of convergence of this series?

   [5 marks]

   (d) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

   [30 marks]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following Mixed homogeneous boundary conditions:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi,\ t > 0 \\ \text{BC} &: \ u(0,t) = 0 = \frac{\partial u(\pi,t)}{\partial x} \\ \text{IC} &: \ u(x,0) = x(\pi - x)\end{aligned}$$

   Hint: It may be useful to know that:
   $\frac{2}{\pi}\int_0^\pi x(\pi - x)\sin(\frac{2n+1}{2}x)dx = \frac{8}{\pi}\frac{4+(-1)^{n+1}(2n+1)\pi}{(2n+1)^3}$

   [30 marks]