Math 257/316, Midterm 2, Section 101

9 am on November 15, 2013

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.
A formula sheet is provided
Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + (1-x)e^{-t},\ 0 < x < 1,\ t > 0 \\ u(0,t) &= 0,\ u(1,t) = e^{-t} \\ u(x,0) &= x + \sin \pi x \end{aligned}$$

by using an appropriate expansion in terms of the apprpriate eigenfunctions.

[60 marks]

2. Consider the following initial boundary value problem for the wave equation:

$$\begin{aligned} u_{tt} &= u_{xx} + 9\pi^2 \sin 3\pi x,\ 0 < x < 1,\ t > 0 \\ u(0,t) &= 0,\ u(1,t) = 0 \\ u(x,0) &= 0,\ u_t(x,0) = \sin \pi x \end{aligned}$$

a) Determine the steady state solution $w(x)$.

b) Let $u(x,t) = w(x) + v(x,t)$ and determine the corresponding boundary value problem for $v(x,t)$.

c) Use the method of separation of variables to solve for $v(x,t)$ and therefore $u(x,t)$.

d) Now use D'Alembert's solution (see the formula sheet) to detemine $v(x,t)$ and therefore $u(x,t)$.

[40 marks]