Math 257/316, Midterm 2, Section 101

9 am on November 15, 2017

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.
A formula sheet is provided
Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + e^{-2t}\cos(x) + x,\ 0 < x < \frac{\pi}{2},\ t > 0 \\ u_x(0,t) &= t,\ u(\frac{\pi}{2}, t) = \frac{\pi}{2}t \\ u(x,0) &= 0 \end{aligned}$$

   by using an expansion in terms of the appropriate eigenfunctions.

   [50 marks]

2. Solve the following initial boundary value problem for the wave equation:

$$\begin{aligned} u_{tt} &= u_{xx},\ 0 < x < \frac{\pi}{2},\ t > 0 \\ u(0,t) &= 0,\ u_x(\frac{\pi}{2}, t) = t \\ u(x,0) &= 0,\ u_t(x,0) = \sin(3x) + x \end{aligned}$$

   Once you have dealt with the inhomogeneous boundary conditions use the method of separation of variables to solve the resulting boundary value problem. Compare your solution to that you obtain using D'Alembert's solution (see the formula sheet).

   [50 marks]