**Math 257/316, Midterm 2, Section 101**

9 am on November 14, 2018

**Instructions.** The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided. **Complete the exam in ink not pencil**. Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + e^{-2t}(\sin 5x - 2x),\ 0 < x < \frac{\pi}{2},\ t > 0 \\ u(0,t) &= 1,\ u_x(\frac{\pi}{2}, t) = e^{-2t} \\ u(x,0) &= 1 + \sin x \end{aligned}$$

   by using an expansion in terms of the appropriate eigenfunctions.

   [50 marks]

   **Hint:** It might be useful to know that: $\int_0^{\pi/2} x \sin((2n-1)x)\, dx = \frac{(-1)^{n+1}}{(2n-1)^2}$

2. Solve the following initial boundary value problem for the wave equation:

$$\begin{aligned} u_{tt} &= u_{xx},\ 0 < x < \pi,\ t > 0 \\ u(0,t) &= \pi t,\ u(\pi,t) = 2\pi t \\ u(x,0) &= \sin 2x,\ u_t(x,0) = \sin x + \pi + x \end{aligned}$$

   Once you have dealt with the inhomogeneous boundary conditions use the method of separation of variables to solve the resulting boundary value problem (do not treat all the cases for the eigenvalue problem). Use D'Alembert's solution (see the formula sheet) to solve the PDE with homogeneous boundary conditions and compare to the solution you obtained by separation of variables.

   [50 marks]