Math 257/316, Midterm 2, Section 102

4 pm on November 15 th, 2013

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + x,\ 0 < x < \pi/2,\ t > 0 \\ u(0,t) &= e^{-\gamma t},\ u_x(\pi/2,t) = t\,,\ \text{where } \gamma > 1 \\ u(x,0) &= 1 + \sin 3x \end{aligned}$$

   by using an appropriate expansion in terms of the apprpriate eigenfunctions.

   [60 marks]

2. Consider the following initial boundary value problem for the wave equation:

$$\begin{aligned} u_{tt} &= u_{xx} - 2,\ 0 < x < 1,\ t > 0 \\ u(0,t) &= 0,\ u(1,t) = 0 \\ u(x,0) &= x^2 - x,\ u_t(x,0) = \sin 2\pi x \end{aligned}$$

   a) Determine the steady state solution $w(x)$.

   b) Let $u(x,t) = w(x) + v(x,t)$ and determine the corresponding boundary value problem for $v(x,t)$.

   c) Use the method of separation of variables to solve for $v(x,t)$ and therefore $u(x,t)$.

   d) Now use D'Alembert's solution (see the formula sheet) to detemine $v(x,t)$ and therefore $u(x,t)$.

   [40 marks]