Math 257/316, Midterm 2, Section 102

4 pm on November 12, 2014

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

A formula sheet is provided

Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + 1,\ 0 < x < \frac{\pi}{2},\ t > 0 \\ u(0,t) &= t,\ u_x(\frac{\pi}{2}, t)\ =\ e^{-\gamma t},\ 0 < \gamma < 1 \\ u(x,0) &= x \end{aligned}$$

by using an appropriate expansion in terms of the appropriate eigenfunctions.

[60 marks]

2. Consider the following initial boundary value problem for the wave equation:

$$\begin{aligned} u_{tt} &= u_{xx} + \gamma \sin(x) + x/\pi,\ 0 < x < \pi,\ t > 0 \\ u(0,t) &= 0,\ u(\pi,t) = t^2/2 \\ u(x,0) &= 0,\ u_t(x,0) = \sin(3x) \end{aligned}$$

a) Determine a simple function $w(x,t)$ that satisfies the inhomogeneous boundary conditions.

b) Let $u(x,t) = w(x,t) + v(x,t)$ and determine the corresponding boundary value problem for $v(x,t)$.

c) Use an eigenfunction expansion (or extract a steady state solution for the boundary value problem for $v(x,t)$ and use separation of variables) to solve for $v(x,t)$ and therefore $u(x,t)$.

[40 marks]

d) **Bonus Marks:** Assuming $\gamma = 0$, use D'Alembert's solution (see the formula sheet) to determine the corresponding $v(x,t)$ and therefore $u(x,t)$.

[5 marks]