Math 257/316, Midterm 2, Section 102

4 pm on November 15, 2017

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.
A formula sheet is provided
Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + e^{-4t}\sin(x) + 1,\ 0 < x < \frac{\pi}{2},\ t > 0 \\ u(0,t) &= t,\ u_x(\frac{\pi}{2}, t) = 1 \\ u(x,0) &= x \end{aligned}$$

by using an expansion in terms of the appropriate eigenfunctions.

[50 marks]

2. Consider the following initial boundary value problem for the damped wave equation with damping coefficient $0 < \gamma < 1$:

$$\begin{aligned} u_{tt} + 2\gamma u_t &= u_{xx},\ 0 < x < \frac{\pi}{2},\ t > 0 \\ u_x(0,t) &= 1,\ u(\frac{\pi}{2}, t) = \frac{\pi}{2} \\ u(x,0) &= x,\ u_t(x,0) = \cos(5x) \end{aligned}$$

a) Determine the steady state solution $w(x)$.

b) Let $u(x,t) = w(x) + v(x,t)$ and determine the corresponding boundary value problem for $v(x,t)$.

c) Use the method of separation of variables to solve for $v(x,t)$ and therefore $u(x,t)$.

[50 marks]