**Math 257/316, Midterm 2, Section 102**

4 pm on November 14, 2018

**Instructions.** The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided. **Complete the exam in ink not pencil**.

Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + e^{-\beta t}\cos 3x - 2,\ 0 < x < \pi,\ t > 0 \\ u_x(0,t) &= \pi,\ u_x(\pi,t) = 3\pi \\ u(x,0) &= \cos x + x^2 \end{aligned}$$

   by using an expansion in terms of the appropriate eigenfunctions. Assume that $\beta > 0$ is not an integer.

   [50 marks]

   **Hint:** It might be useful to know that: $\int\limits_0^\pi x\cos nx dx = \frac{(-1)^n - 1}{n^2}$

2. Solve the following initial boundary value problem for the wave equation describing the dymanics of a string resting on an elastic foundation having a stiffness $\gamma \geq 0$:

$$\begin{aligned} u_{tt} &= u_{xx} - \gamma u,\ 0 < x < \pi,\ t > 0 \\ u(0,t) &= 0,\ u(\pi,t) = 0 \\ u(x,0) &= \sin x,\ u_t(x,0) = \sin 2x \end{aligned}$$

   Use the method of separation of variables to solve the resulting boundary value problem (do not treat all the cases for the eigenvalue problem). Now set $\gamma = 0$ and compare your solution to that you obtain using D'Alembert's solution (see the formula sheet).

   [50 marks]