**Math 257/316, Midterm 2, Section 201**

11 am on March 19, 2012

**Instructions.** The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} + x,\ 0 < x < 1,\ t > 0 \\ u(0,t) &= 1 - e^{-t},\ u_x(1,t) = t \\ u(x,0) &= 0 \end{aligned}$$

   by using an appropriate expansion in terms of the eigenfunctions $\sin\left(\frac{(2n+1)\pi}{2}x\right)$ corresponding to the eigenvalues $\lambda_n = \frac{(2n+1)\pi}{2}$ where $n = 0, 1, \ldots$.

   [60 marks]

2. Solve the following initial boundary value problem for the wave equation:

$$\begin{aligned} u_{tt} &= c^2 u_{xx},\ 0 < x < 1,\ t > 0 \\ u_x(0,t) &= 0,\ u(1,t) = 0 \\ u(x,0) &= x,\ u_t(x,0) = 0 \end{aligned}$$

   either by using the method of separation of variables or the appropriate eigenfunction expansion. Interpret your solution in terms of the D'Alembert solution (see the formula sheet).

   [40 marks]