Math 406, Midterm
4 November 2013

Instructions. The duration of the exam is 55 minutes. Answer all questions. A single page of A4 notes written on both sides of the page is permitted into the exam.

Maximum score 50.

1. (a) Use the folllowing error estimate for interpolation of a smooth function $f(x)$ by a polunomial $p_N(x)$ of degree N that passes through the $N+1$ points $x_0, \ldots, x_N$

$$f(x) = p_N(x) + \frac{f^{(N+1)}(\xi)}{(N+1)!}(x - x_0)\ldots(x - x_N)$$

to determine an estimate for the local truncation error for the Trapezium rule. Use this result to obtain an error estimate for the composite Trapezium Rule.

(b) Now estimate the integral

$$I = \int_0^1 e^{-x} dx = 1 - e^{-1}$$

by means of the composite Trapezium Rule with $h = 1/2$ and $h = 1/4$.

(c) Use these estimates along with Richardson Extrapolation to determine a better estimate for I.

[20 marks]

2. Consider the boundary value problem

$$Lu = x^2u'' - xu' - 3u = f,\ u(0) < \infty \text{ and } u(1) = 4 \tag{1}$$

(a) Determine the adjoint operator L^* and appropriate boundary conditions associated with L and the boundary conditions above. Determine the Green's function G so that u has an integral representation in terms of G and f.

(b) Multiply the equation in (1) by an appropriate function to make L formally self-adjoint. Now determine the Green's function for the new problem and use this to obtain an integral representation for u in terms of f. How does this compare to the integral representation found in part (a)?

[30 marks]