Math 257/316 Assignment 3
Due Monday January 30th in class

Problem 1: Find all singular points of the followings equations and determine whether each of them is regular or irregular. For each regular singular point, determine the indicial equation and the exponents at the singularity.

1. $-2(x-1)^2(x^2-2)^3y'' + 2(x^2-1)y' - 4xy = 0$
2. $2\tan^2 xy'' - \sin xy' - y = 0$

Problem 2: For the following equation, verify that $x = 0$ is a regular singular point, find the indicial equation and the exponents at the singularity, find the recurrence relation and find the first three nonzero terms of the series solution for $x > 0$ corresponding to the larger root of the indicial equation.

$$4x^2y'' + 4x(x+1)y' - y = 0$$

Problem 3: Verify that $x = 0$ is a regular singular point of the following equation. Find the first three nonzero terms in each of two linearly independent solutions about $x = 0$.

$$8x^2y'' - 2xy' + 3(x+1)y = 0$$

Problem 4: The Chebyshev equation: $(1-x^2)y'' - xy' + \alpha y = 0$ has regular singular points at $x = \pm 1$ and an ordinary point at $x = 0$.

a) Determine two independent Taylor Series solutions around $x = 0$. What happens when α is an integer?

b) Determine the indicial equation and the exponent at the singularity $x = 1$.

b) Assume $\alpha \neq 0, 1$. Find the first three nonzero terms of the series solution in powers of $(x-1)$.(Hint: Write $(1-x^2) = -(x-1)(x+1) = -(x-1)(2+(x-1))$ and $x = 1 + (x-1)$).

c) What happens if $\alpha = 0$ or $\alpha = 1$?