University of British Columbia
Math 301, Section 201

Midterm 1

Date: February 13, 2012
Time: 11:00 - 11:50pm

Name (print):
Student ID Number:
Signature:

Instructor: Richard Froese

Instructions:

1. No notes, books or calculators are allowed.
2. Read the questions carefully and make sure you provide all the information that is asked for in the question.
3. Show all your work. Answers without any explanation or without the correct accompanying work could receive no credit, even if they are correct.
4. Answer the questions in the space provided. Continue on the back of the page if necessary.

Question	Mark	Maximum
1		12
2		12
3		16
Total		40

[4] 1. (a) Evaluate $I = \int_0^\infty \frac{1}{(x^2+1)^2}\,dx$.

[4] (b) Evaluate $I = \int_0^{2\pi} \frac{1}{2\cos(\theta)+3}\,d\theta$.

[4] (c) Evaluate $I = \oint_{|z|=1} z^n e^{1/z} dz$ for all $z \in \mathbb{Z}$, where the contour is traversed in the counter-clockwise direction.

[4] 2. (a) Identify the branch points of the multivalued function $\log((z+1)/(z-1))$. Don't forget to check $z = \infty$.

[4]

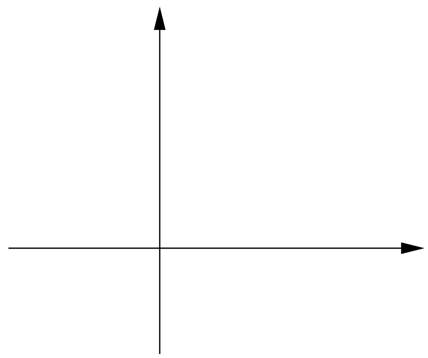
(b) If a single valued branch of the function above is defined as $\mathrm{Log}((z+1)/(z-1))$ (using the principal branch of the logarithm) where are the branch cuts?

[4]

(c) Evaluate $I = \oint_{|z|=3} (z-2)^{-1}\mathrm{Log}((z+1)/(z-1))dz$ where the contour is traversed in the counterclockwise direction.

3. Suppose that you wish to calculate $I = \int_0^\infty \frac{x^\alpha}{x^4+16}\,dx$ by integrating $f(z) = \frac{z^\alpha}{z^4+16}$ around a suitable contour and taking limits.

[2] (a) Sketch the contour you would use to evaluate this integral.



[3] (b) Specify the branch of z^α that you would use by describing by the range of angles method how you would compute your branch of z^α for every $z \in \mathbb{C}$. Draw the branch cut on your sketch.

[2] (c) Are there other branches of z^α that would work in your integral? If so, describe them.

[2] (d) Find and classify the isolated singular points inside your contour. Draw them onto your sketch. Calculate the residue at these points.

[3] (e) For each piece of the contour where you want the integral to vanish in the limit, indicate the range of α where this happens, and provide an explicit estimate to prove it.

[3]

(f) Calculate the integral over the remaining pieces of your contour, and use the result to find I. For what range of α is your formula valid.

[1]

(g) Write I in a pretty form involving only real quantities.