University of British Columbia
Math 301, Section 201

Midterm 2

Date: March 19, 2012
Time: 11:00 - 11:50pm

Name (print):
Student ID Number:
Signature:

Instructor: Richard Froese

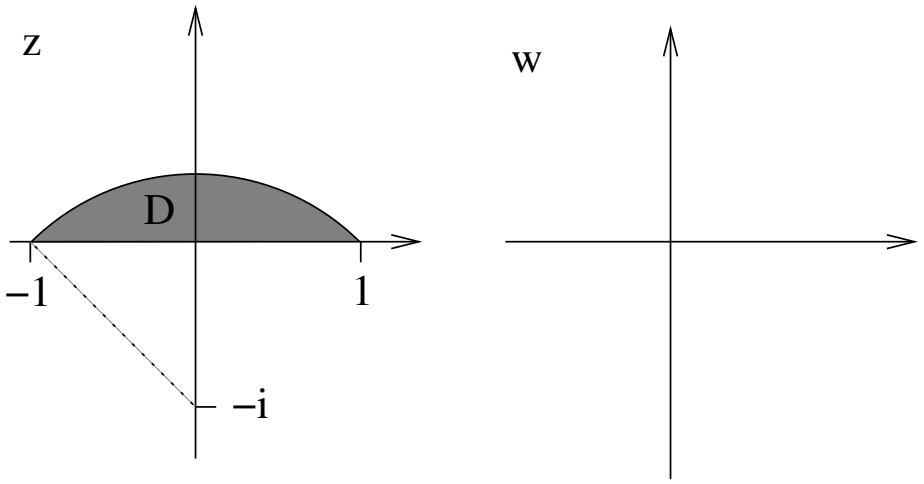
Instructions:

1. No notes, books or calculators are allowed.
2. Read the questions carefully and make sure you provide all the information that is asked for in the question.
3. Show all your work. Answers without any explanation or without the correct accompanying work could receive no credit, even if they are correct.
4. Answer the questions in the space provided. Continue on the back of the page if necessary.

Question	Mark	Maximum
1		15
2		8
3		9
4		8
Total		40

[5] 1. (a) Write down the fractional linear transformation $f(z)$ satisfying $f(-1) = 0$, $f(0) = 1$ and $f(1) = \infty$.

Let D be the region depicted below. (D is bounded below by the segment $[-1, 1]$ on the real line and above by an arc of the circle centred at $-i$ going through -1 and 1.)



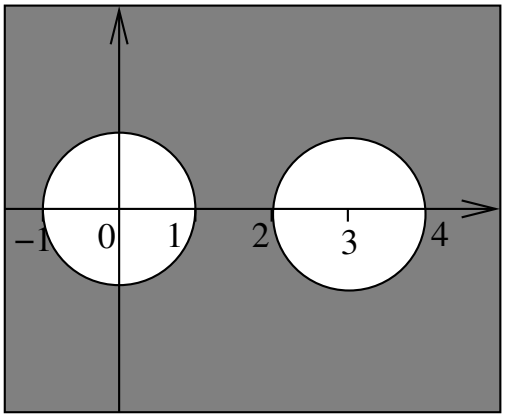
[3] (b) What is the image of the top (circular) part of the boundary of D under $w = f(z)$? Give a complete explanation of your answer.

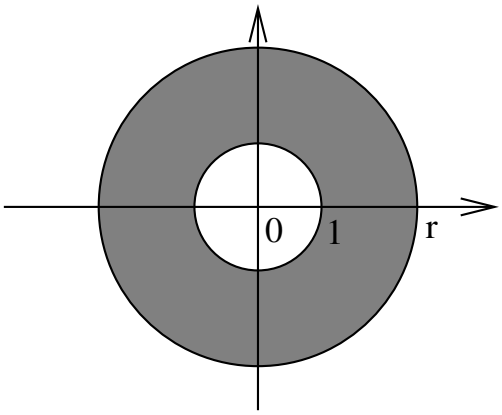
[2] (c) Draw the image of the region D the map $w = f(z)$ on the diagram above.

[5] (d) Solve Laplace's equation $\Delta\varphi(x,y) = 0$ for $z = x + iy \in D$ with boundary conditions $\varphi(x,0) = 1$ for $-1 \leq x \leq 1$ and $\varphi(x,y) = 0$ for $x + iy$ on the circular portion of the boundary.

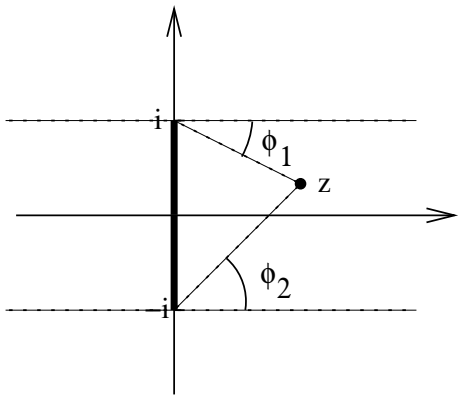
[8]

2. Find a conformal map that maps the region outside the circles $\{|z| \leq 1\}$ and $\{|z-3| \leq 1\}$ (depicted on the left) to the annulus $1 < |z| < r$ (depicted on the right). What is the value of r?





3. Consider the complex velocity potential $\Omega(z) = v_0(z^2+1)^{1/2}$, $v_0 > 0$, where the branch is chosen so that $\Omega(z)$ has a branch cut on the imaginary axis between $-i$ and i. Concretely, $\Omega(z) = v_0|z-i|^{1/2}|z+i|^{1/2}\exp((\phi_1+\phi_2)/2)$ in terms of the angles ϕ_1 and ϕ_2 depicted below, with $\phi_1, \phi_2 \in [-\pi/2, 3\pi/2]$.



[3]

(a) How does the complex fluid velocity behave as $|z| \to \infty$?

[3]

(b) Is the set $\mathbb{R}\backslash\{0\} = \{z = x + iy : y = 0, x \neq 0\}$ a streamline for this flow? Give a reason.

[3]

(c) Show that this potential represents idealized inviscid fluid flow around a thin plate positioned on the branch cut.

[8]

4. How many zeros does $p(z) = z^5 + z^3 + z + 1$ have in the right half plane?