University of British Columbia
Math 301, Section 201

Midterm 2

Date: March 15, 2013
Time: 11:00 - 11:50pm

**Name (print):**
Student ID Number:
Signature:

Instructor: Richard Froese

Instructions:

1. No notes, books or calculators are allowed.

2. Read the questions carefully and make sure you provide all the information that is asked for in the question.

3. Show all your work. Answers without any explanation or without the correct accompanying work could receive no credit, even if they are correct.

4. Answer the questions in the space provided. Continue on the back of the page if necessary.

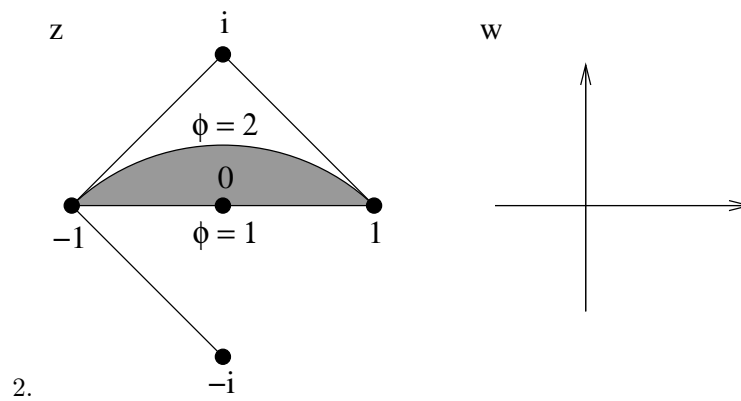| Question	Mark	Maximum
1		12
2		12
3		16
Total		

z

−1 + i

1 + i

−1 − i

1 − i

1.

L

w

0

1

[4]
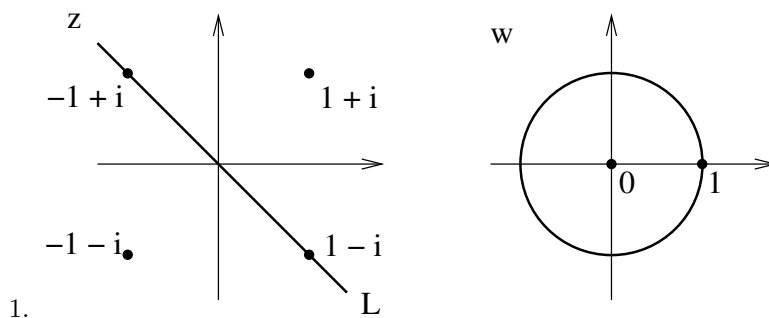
(a) Suppose that $f(z)$ is a fractional linear transformation that

   i. maps the line L passing through $-1+i$ and $1-i$ to the unit circle,

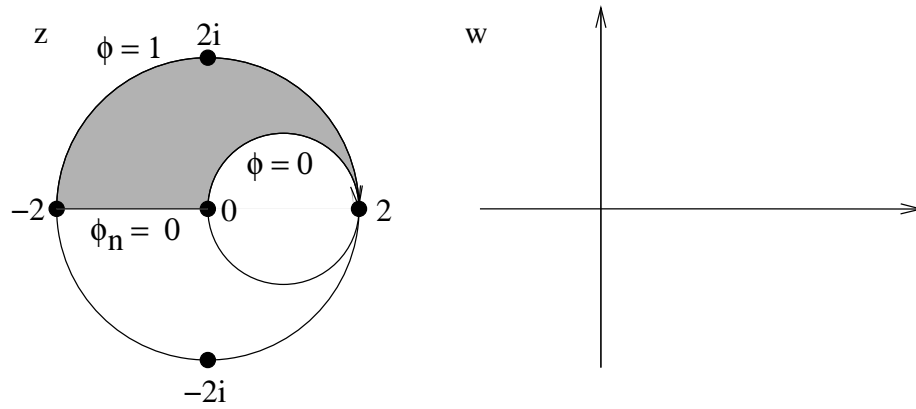   ii. maps $1+i$ to 0.

What is $f(-1-i)$? Give a reason.

[8]

(b) Find all fractional linear transformations satisfying conditions i. and ii. above. If $f(0) = 1$ what is $f(1-i)$?

z   i   w

$\phi = 2$

0

-1 $\phi = 1$ 1

2. $-i$

[12]

(a) Solve Laplace's equation $\Delta\phi = 0$ in the shaded region (above the segement $[-1, 1]$ and below the circle $|z + i| = \sqrt{2}$) with the indicated boundary conditions.

z $\phi = 1$ 2i w

$\phi = 0$

−2 0 2

$\phi_n = 0$

3. −2i

[4]

(a) Determine the image of the shaded region under $f(z) = -(z+2)/(z-2)$ and draw it on the w plane above

[8]

(b) Solve Laplace's equation in the shaded region with the indicated boundary conditions. In the diagram, ϕ_n denotes the normal derivative $\partial\phi/\partial n$.

[4]

(c) The conformal map $g(w) = \sin(\pi w/2)$ maps the image region from part (a) to the first quadrant. Explain how you could use this additional map to solve Laplace's equation above with the boundary condition on the segment $[-2, 0]$ (i.e., $\phi_n = 0$) replaced by $\phi = 0$. You need not carry out the calculations in detail.