University of British Columbia
Math 301

Midterm 2

Date: March 20, 2020
Time: 11:00 - 12:20pm

Instructions:

1. Notes and books are ALLOWED. Internet access (other than for downloading/printing and submitting to canvas) and communication with others NOT ALLOWED.
2. Show all your work and explain what you are trying to do.
3. Use your own paper to write the answers. Make sure your student number is on each sheet.
4. Please write out and sign the following pledge at the end of your answers.

   I affirm that I did not give or receive any unauthorized help on this examination, that all work is my own.

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1. How many zeros does the polynomial $p(z) = 1 + z + \frac{5}{2}z^2 + z^3 + z^4$ have
   (a) (10 points) in the right half plane $\{z : \mathrm{Re}(z) > 0\}$
   (b) (5 points) in the upper half plane $\{z : \mathrm{Im}(z) > 0\}$
   (c) (5 points) in the second quadrant $\{z : \mathrm{Re}(z) < 0, \mathrm{Im}(z) > 0\}$

   Explain why the procedure you are using is counting the zeros. You may use the fact that $p(x)$ does not vanish for $x \in \mathbb{R}$.

2. Does there exist a Mobius transformation f that maps the real axis onto the unit circle and satisfies
   (a) (5 points) $f(i) = 2$ and $f(-i) = -1/2$?
   (b) (5 points) $f(i) = 2$ and $f(-i) = 1/2$?

   In each part, give a reason (if the answer is no) or find such an f (if the answer is yes).

3. Find a solution to $\Delta\varphi(x, y) = 0$ in the upper half plane and outside the unit circle satisfying the boundary conditions $\varphi(x, 0) = 0$ for $x < -1$ and for $x > 1$, and $\varphi(x, y) = 1$ for $x^2 + y^2 = 1$ and $y \geq 0$.