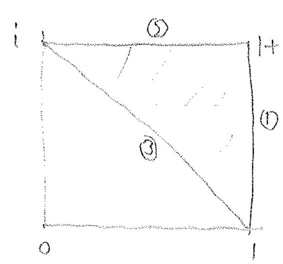
Math 301 Hmk 6 Solutions

1. Find the image of the triangle with vertices 1, i, $1+i$ under the map $f(z) = z^2$

Solution: Map the boundary pieces: Let $z = x+iy$



$w = f(z) = (x+iy)^2 = x^2 - y^2 + 2ixy = u + iv$

On ①, $x = 1$ $y \in [0,1]$ $u = 1-y^2$, $v = 2y$
The image lies on the parabola $u = 1 - \frac{1}{4}v^2$ between $(u,v) = (1,0)$ and $(u,v) = (0,2)$.

On ② $y = 1$ and $x \in [0,1]$ so $u = x^2 - 1$, $v = 2x$. The image lies on the parabola $u = \frac{1}{4}v^2 - 1$ beteen $(u,v) = (-1,0)$ and $(0,2)$.

On ③ $x + y = 1$ so $y = 1 - x$ and $x \in [0,1]$. So $u = x^2 - (1-x)^2 = x^2 - 1 + 2x - x^2 = 2x - 1$ and $v = 2xy = 2x(1-x)$; Thus
$x = (u+1)/2$ so $v = 2\left(\frac{u+1}{2}\right)\left(1 - \frac{u+1}{2}\right) = (u+1)\left(\frac{1-u}{2}\right) = \frac{1-u^2}{2}$ So
(u,v) lies on the parabola $v = \frac{1}{2} - \frac{u^2}{2}$,

Image is