

Fourier Transform Summary

	$f(x)$	$\widehat{f}(k)$
Definition and inversion		
	$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \widehat{f}(k) dk$	$\int_{-\infty}^{\infty} e^{-ikx} f(x) dx$
Examples		
1	$\frac{1}{1+x^2}$	$\pi e^{- k }$
2	$e^{- x }$	$\frac{2}{1+k^2}$
3	$\begin{cases} 1 & x < 1 \\ 1/2 & x = 0 \\ 0 & x \geq 1 \end{cases}$	$2 \frac{\sin(k)}{k}$
4	$\frac{\sin(x)}{x}$	$\pi \begin{cases} 1 & k < 1 \\ 1/2 & k = 0 \\ 0 & k > 1 \end{cases}$
5	$e^{-x^2/(2\sigma^2)}$	$\sqrt{2\pi} \sigma e^{-\sigma^2 k^2/2}$
Properties		
0	$c_1 f_1(x) + c_2 f_2(x)$	$c_1 \widehat{f}_1(k) + c_2 \widehat{f}_2(k)$
1	$f(x+a)$	$e^{iak} \widehat{f}(k)$
2	$e^{iax} f(x)$	$\widehat{f}(k-a)$
3	$f(ax)$	$a^{-1} \widehat{f}(k/a)$
4	$f'(x)$	$ik \widehat{f}(k)$
5	$ixf(x)$	$-\widehat{f}'(k)$
6	$f * g(x)$	$\widehat{f}(k) \widehat{g}(k)$
7	$f(x)g(x)$	$(2\pi)^{-1} \widehat{f} * \widehat{g}(k)$

Note about the examples 3 and 4: The value of a Fourier or inverse Fourier transform is insensitive to changes of the input function at a single point. In property 6: $f * g(x) = \int_{-\infty}^{\infty} f(x-y)g(y)dy$

Laplace Transform Summary

	$y(t)$	$Y(s)$
Definition and inversion		
	$\frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{st} Y(s) ds$	$\int_0^{\infty} e^{-st} y(t) dt$
Examples		
1	e^{-at}	$\frac{1}{s+a}$
2	$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
3	$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
Properties		
0	$c_1 y_1(t) + c_2 y_2(t)$	$c_1 Y_1(s) + c_2 Y_2(s)$
1	$y'(t)$	$sY(s) - y(0)$
2	$ty(t)$	$-Y'(s)$
3	$e^{at} y(t)$	$Y(s-a)$
4	$u(t-a)y(t-a)$	$e^{-ac} Y(s)$
5	$y_1 * y_2(t)$	$Y_1(s)Y_2(s)$

In property 4: $u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$. In property 5:
 $y_1 * y_2(t) = \int_0^t y_1(t-\tau)y_2(\tau)d\tau$