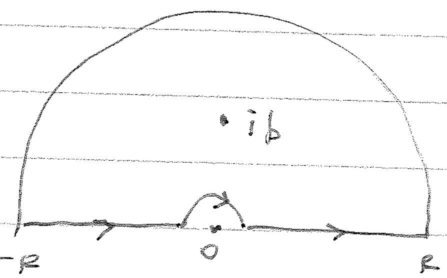
$\boxed{6 \cdot 5 : 12}$ $\quad I = \int_0^\infty \frac{\sin(ax)}{x(x^2+b^2)}\, dx$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \frac{\sin(ax)}{x(x^2+b^2)}\, dx$$

$$= \frac{1}{2}\ \text{p.v.} \int_{-\infty}^{\infty} \frac{\sin(ax)}{x(x^2+b^2)}\, dx$$

$$= \frac{1}{2}\ \mathrm{Im}\ \text{p.v.} \int_{-\infty}^{\infty} \frac{e^{iax}}{x(x^2+b^2)}\, dx$$



Using the contour, $\text{p.v.} \int_{-\infty}^{\infty} \frac{e^{iax}}{x(x^2+b^2)}\, dx$

$$- i\pi \operatorname{Res}\left[\frac{e^{iaz}}{z(z^2+b^2)} : z=0\right] = 2\pi i \operatorname{Res}\left[\frac{e^{iaz}}{z(z^2+b^2)} ; z=ib\right]$$

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{e^{iaz}}{z(z^2+b^2)} = i\pi\left(\frac{1}{b^2}\right) + 2\pi i \frac{e^{-ab}}{ib(\cdot 2ib)}$$

$$= \frac{i\pi}{b^2} + \frac{2\pi i\, e^{-ab}}{-2b^2}$$

$$= \frac{i\pi}{b^2}\left(1 - e^{-ab}\right)$$

So $I = \frac{1}{2}\mathrm{Im}\,(\cdot\cdot) = \frac{\pi}{2b^2}\left(1 - e^{-ab}\right)$