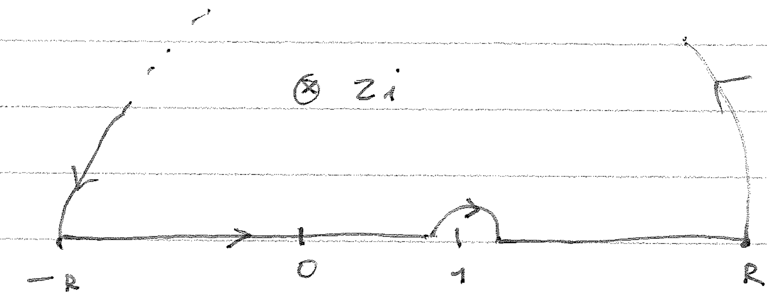
6.5 : 6, 9, 12

$\boxed{6.5 : 6}$ $\quad$ $\text{p.v.} \displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{(x^2+4)(x-1)}\, dx$

Integrate $\dfrac{e^{iz}}{(z^2+4)(z-1)}$ around the indented contour



This yields

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{e^{ix}}{(x^2+4)(x-1)}\, dx \; - \; i\pi \, \text{Res}\left[\frac{e^{iz}}{(z^2+4)(z-1)} \, ; \, z=1 \right]$$

$$= 2\pi i \; \text{Res}\left[\frac{e^{iz}}{(z^2+4)(z-1)} \, ; \, z = 2i \right]$$

Both are simple poles. So

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{e^{ix}}{(x^2+4)(x-1)}\, dx = i\pi \frac{e^{iz}}{5} + 2\pi i \frac{e^{-2}}{2z(z-1)}\Big|_{z=2i}$$

$$= \frac{i\pi}{5} e^{iz} + \frac{2\pi i \, e^{-2}}{4i(2i-1)}$$

$$= \frac{i\pi}{5} e^{i} + \frac{\pi}{2} \cdot \frac{-2i-1}{5} e^{-2}$$

Now take Im part to get

$$\int_{-\infty}^{\infty} \frac{\sin(x)}{(x^2+4)(x-1)}\,dx = \frac{\pi}{5}\cos(1) - \frac{\pi}{5}e^{-2}$$

$\boxed{6.5:9}$

Consider $f(z) = \frac{3}{4}\left(e^{iz}-1\right) - \frac{1}{4}\left(e^{3iz}-1\right)$

Expanding at $z=0$ gives $f(z) = \frac{3}{4}\Big(iz + \frac{1}{2}(iz)^2$

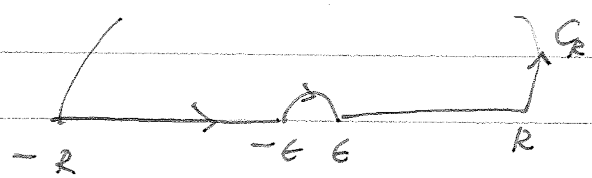
$+ O(z^3)\Big) - \frac{1}{4}\left(3iz + \frac{1}{2}(3iz)^2 + O(z^3) \right)$.

$= \frac{3iz}{4} - \frac{3z^2}{8} - \frac{3iz}{4} + \frac{9z^2}{8} + O(z^3)$

$= \frac{3z^2}{4} + O(z^3)$. Thus $\frac{f(z)}{z^3} = \frac{3}{4z} + O(1)$

has a simple pole at $z=0$ with residue $\frac{3}{4}$.

Now we integrate $\frac{f(z)}{z^3}$ around.



and take $R \to \infty$ and $\epsilon \to 0$. Notice that

for $\operatorname{Im}(z) \geq 0$ $|f(z)| \leq \frac{3}{4}(2) + \frac{1}{4}(2) = 2$

so $\frac{|f(z)|}{|z^3|} \leq \frac{2}{|z|^3}$. This implies $\lim_{R\to\infty} \int_{C_R} \frac{f(z)}{z^3}\,dz = 0$

So we find $\text{p.v.} \int_{-\infty}^{\infty} \frac{f(z)}{z^3}\,dz = i\pi \operatorname{Res}\left[\frac{f(z)}{z^3}, 0\right]$

$= \frac{3i\pi}{4}$. Take Im to get $\text{p.v.} \int \frac{\sin^3(x)}{x^3}\,dx = \frac{3\pi}{4}$