Correction to: Upper bounds for the resonance counting function

of Schrödinger operators in odd dimensions

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The proof of Lemma 3.4 relies on the incorrect equality $\mu_j(AB) = \mu_j(BA)$ for singular values (for a counterexample, see [S], p. 4.) Thus, Theorem 3.1 as stated has not been proven. However, with minor changes, we can obtain a bound for the counting function in terms of the growth of the Fourier transform of $|V|$. The author thanks Barry Simon for pointing out this error.

Here is the corrected version of Theorem 3.1.

Theorem *Suppose that V is a super-exponentially decaying potential with*

$$\widehat{|V|}(z) \leq Ce^{\Phi(|z|)}$$

for a positive, increasing function Φ. Then

$$n(r) \leq C\Phi^n(cr) + O(\Phi^{n-1}(cr))$$

for some constants c and C.

These are the changes needed to prove the bound for $|\phi(k)|$ in Lemma 3.4. Using $\det(1+AB) = \det(1+BA)$ and Fan's inequality $\mu_{n+m+1}(AB) \leq \mu_{n+1}(A)\mu_{m+1}(B)$ (see [S]) we arrive at

$$\mu_j(T(k)) \leq C|k|^{n-2}\mu_{[(j+1)/2]}(F_V^T(-k))\mu_{[(j+1)/2]}(F_{|V|}(-k)) \qquad ()$$

where $[\cdot]$ denotes the integer part. Now

$$\mu_{[(j+1)/2]}(F_{|V|}(-k)) = \left(\mu_{[(j+1)/2]}\mathbf{V}_k)\right)^{1/2}$$

where this time $\mathbf{V}_k$ is the integral operator with integral kernel $\widehat{|V|}(\overline{k}\omega - k\omega')$. We then obtain the bound

$$\mu_{[(j+1)/2]}(F_{|V|}(-k)) \leq Ce^{(\Phi-\delta[(j+1)/2]^{(1/(n-1))})/2}$$

and the same bound for $\mu_{[(j+1)/2]}(F_V^T(-k))$. This leads to

$$\mu_j(T(k)) \leq Ce^{\Phi-\delta' j^{1/(n-1)}}$$

where $\Phi = \Phi((2+\epsilon)|k|)$ for some $\epsilon > 0$ and $\delta' = \delta 2^{-1/(n-1)}$. The rest of the proof is identical.