1. Find an approximation for $\int_0^{0.5} \frac{1}{1+x^4}\, dx$ good to 20 decimal places.

   Solution:
   Substituting $-x^4$ for x in the standard Maclaurin series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots, \quad -1 < x < 1$$

   gives

$$\frac{1}{1+x^4} = 1 - x^4 + x^8 - x^{12} + - \cdots, \quad -1 < x < 1$$

   Integrating term by term gives

$$\begin{aligned}\int_0^{0.5} \frac{1}{1+x^4}\, dx &= \int_0^{0.5} (1 - x^4 + x^8 - x^{12} + - \cdots)\, dx \\ &= (x - x^5/5 + x^9/9 - x^{13}/13 + - \cdots)\big|_0^{0.5} \\ &= 0.5 - (0.5)^5/5 + (0.5)^9/9 - (0.5)^{13}/13 + - \cdots\end{aligned}$$

   We interpret "to 20 decimal places" as "with error $< 0.5 \cdot 10^{-20}$", so we need the first omitted term to be less than this value. Using a calculator, we check that $(0.5)^{61}/61 > 0.5 \cdot 10^{-20}$, but $(0.5)^{65}/65 < 0.5 \cdot 10^{-20}$, so we must use all terms up to and including $(0.5)^{61}/61$. (If we interpret "to 20 decimal places" as "with error $< 10^{-20}$", then we only need to go up to $(0.5)^{57}/57$.)