

Some rules of differentiation

DEFINITION:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{y \rightarrow x} \frac{f(y) - f(x)}{y - x}$$

1. LINEARITY:

$$(f + \alpha g)'(x) = f'(x) + \alpha g'(x) \quad (\alpha \in \mathbb{R} \text{ constant})$$

2. PRODUCT RULE:

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x).$$

3. QUOTIENT RULE:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}, \quad (g(x) \neq 0)$$

4. CONSEQUENCES:

(i) Power rule:

$$f(x) = x^m \quad (m \in \mathbb{N}) \quad \implies \quad f'(x) = mx^{m-1}$$

(ii) Derivative of $\frac{1}{f}$

$$\left(\frac{1}{f}\right)'(x) = -\frac{f'(x)}{(f(x))^2}$$

5. CHAIN RULE:

$$(f \circ g)'(x) = f'(g(x))g'(x)$$