MID TERM #1, MATH 100

Wednesday, October 9, 2002

Student No: _____ Name (Print): _____

There are 5 pages to this test, check to make sure it is complete. **Please put your name and student number at the top of every page.** Rough work should be done on the backs of the pages. **You must show all your work to get full marks. Calculators and notes of any kind are not allowed.**

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1. [6 marks] Using only the definition of the derivative, and not the rules, find $f'(x)$ for the function $f(x) = \sqrt{x^2 + 1}$.

Please do not write in this space.

Number	Value	Grade
Question 1	6	
Question 2	12	
Question 3	8	
Question 4	8	
Question 5	6	
Question 6	10	
Total	50	

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2. [12 marks] Find the derivatives of the following functions. Put your answers in the boxes and show your work in the spaces provided. **DO NOT SIMPLIFY YOUR ANSWERS.**

(a) $f(x) = \left(\sin^3 x + \cos^3 x\right)^2$.

(b) $f(x) = \sqrt{1 + \sqrt{x + x^2}}$.

(c) $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$.

(d) $f(x) = (x^2 + x + 1)(x^3 + 1)$.

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3. [8 marks]

(a) Determine $\lim_{\theta \to 0} \dfrac{\tan 2\theta}{\theta}$.

(b) Find the absolute maximum and minimum of the function $f(x) = x^2 + \dfrac{1}{x^2}$ on the interval $\dfrac{1}{2} \leq x \leq 3$.

(c) Find all x where the derivative of $y = \sin x + \cos x$ is 0.

(d) Determine $\lim_{x \to 1} \dfrac{1}{x-1}\left(\dfrac{1}{\sqrt{x}} - 1\right)$.

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4. [8 marks] Find the dimensions of the largest rectangle which can be inscribed in a right triangle with sides 6, 8 and 10. Assume 2 sides of the rectangle lie on the legs of the right triangle.

5. [6 marks] A cannon ball is shot vertically upwards from the ground with initial velocity $v_o = 98m/s$. It is determined that the height of the ball, $y(t)$ (in meters), as a function of the time, t (in sec), is given by $y = v_o t - 4.9t^2$.

(a) When does the cannon ball reach its highest point?

(b) At what time does the cannon ball hit the ground?

(c) What is the velocity of the cannon ball when it hits the ground?

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6. [10 marks] Let $f(x)$ be the function defined by $f(x) = \begin{cases} x^2 \cos(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(a) Compute $f'(x)$ for $x \neq 0$.

(b) Prove that $f'(0) = 0$.

(c) Prove that $f'(x)$ is not continuous at $x = 0$.