MID TERM #2, MATH 100

Wednesday, November 13, 2002

Student No: _____ Name (Print): _____

There are 5 pages to this test, check to make sure it is complete. **Please put your name and student number at the top of every page.** Rough work should be done on the backs of the pages, and **your answers put in the boxes (if provided). You must show all your work to get full marks. Calculators and notes of any kind are not allowed.**

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1. [6 marks]

(a) Find the derivative of $f(x) = \arcsin(\sqrt{x})$. Do not simplify.

(b) Find $\dfrac{f'(x)}{f(x)}$ if $f(x) = (\ln x)^x$ and simplify.

(c) Find $f'(x)$ for $f(x) = \arctan\left(\dfrac{x-1}{x+1}\right)$ and simplify.

Please do not write in this space.

Number	Value	Grade
Question 1	6	
Question 2	8	
Question 3	12	
Question 4	12	
Question 5	12	
Total	50	

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2. [8 marks] Let $f(x)$ be the function $f(x) = x\,(\ln x)^2\,,\ x > 0$.

(a) Find all x where $f'(x) = 0$.

(b) Find all x where $f''(x) = 0$.

(c) Find all intervals where $f(x)$ is decreasing.

(d) Find all intervals where $f''(x) < 0$.

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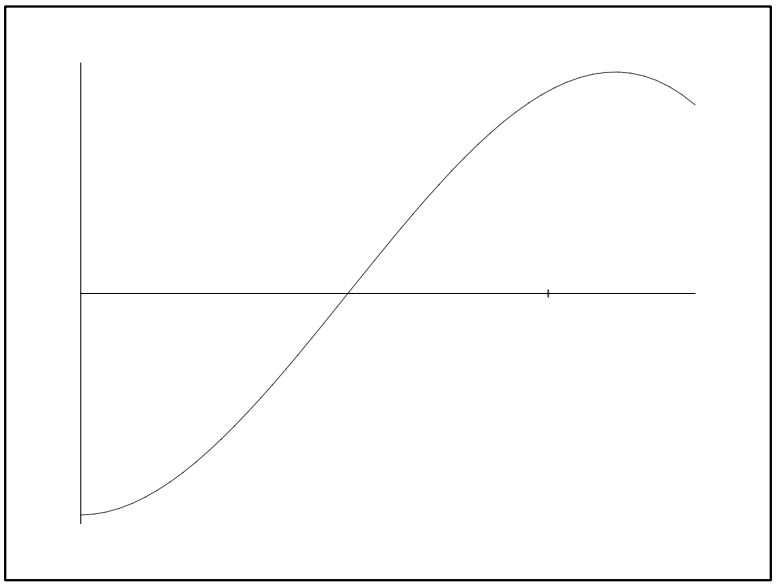
3. [12 marks]

(a) Suppose $f(x)$ is defined for all x and satisfies $f'(x) = \dfrac{x}{1+x^2}$, $f(1) = 2$. Use a linear approximation to estimate $f(0.99)$.

(b) Locate on the graph below the approximations x_1, x_2 resulting from Newton's method, if the starting value is x_0.



(c) State the Mean Value Theorem.

(d) Prove that $1 + \dfrac{x}{2} \geq \sqrt{1+x}$ for all $x \geq 0$.

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4. [12 marks] At time $t = 0$ a pot of boiling water is removed from a stove into a room where the ambient temperature is 20^o Celsius. 10 minutes later its temperature is 60^o Celsius. Assume the temperature of boiling water is 100^o Celsius.

(a) Determine a formula for the temperature $T(t)$ of the water at any time t.

(b) What will the temperature be after 20 minutes (that is at $t = 20$).?

Show your work here.

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5. [12 marks] Let $y = f(x)$ be the function defined implicitly near $x = 1$ by

$$x^3 - xy + y^3 = 1, \ y = 1 \text{ when } x = 1.$$

(a) Find y' for x near 1.

(b) Is $f(x)$ increasing or decreasing near $x = 1$? You must give a cogent reason for your assertion.

(c) Determine an equation for the tangent line of $y = f(x)$ at $x = 1$.

(d) Use a linear approximation to estimate $f(1.1)$.