# SOLUTIONS TO HOMEWORK ASSIGNMENT #1

1. Compute the following limits:

(a) $\lim_{x\to -2}(x^3-3x^2+5)$ (b) $\lim_{x\to -1}\dfrac{(3x^2+2x+1)^{10}}{(x^3+5)^5}$ (c) $\lim_{t\to 2}(-3t^3-4t+5)^{1/3}$

(d) $\lim_{t\to 3}\dfrac{t^2-9}{t-3}$ (e) $\lim_{t\to -3}\dfrac{t^3-9t}{t+3}$ (f) $\lim_{z\to 9}\dfrac{3-\sqrt{z}}{9-z}$

SOLUTIONS:

(a) $\lim_{x\to -2}(x^3-3x^2+5)=(-2)^3-3(-2)^2+5=-15$ by substituting $x=-2$ and using continuity of the polynomial $x^3-3x^2+5$ at $x=-2$.

(b) $\lim_{x\to -1}\dfrac{(3x^2+2x+1)^{10}}{(x^3+5)^5}=\dfrac{(3(-1)^2+2(-1)+1)^{10}}{((-1)^3+5)^5}=\dfrac{2^{10}}{4^5}=1$ by substituting $x=-1$ and using continuity of the function $\dfrac{(3x^2+2x+1)^{10}}{(x^3+5)^5}$ at $x=-1$.

(c) $\lim_{t\to 2}(-3t^3-4t+5)^{1/3}=(-3(2)^3-4(2)+5)^{1/3}=(-27)^{1/3}=-3$ by substituting $t=2$ and using continuity of the function $(-3t^3-4t+5)^{1/3}$ at $t=2$.

(d) $\lim_{t\to 3}\dfrac{t^2-9}{t-3}=\lim_{t\to 3}\dfrac{(t-3)(t+3)}{t-3}=\lim_{t\to 3}(t+3)=6$. Notice that we can not put $t=3$ right away as it leads to the nonsense $\left[\dfrac{0}{0}\right]$. This is why we must first do some algebra, namely cancel $t-3$ from both denominator and numerator. Once this is done we can evaluate the limit by putting $t=3$ and using continuity of the linear function $t+3$.

(e) $\lim_{t\to -3}\dfrac{t^3-9t}{t+3}=\lim_{t\to -3}\dfrac{t(t-3)(t+3)}{t+3}=\lim_{t\to -3}t(t-3)=18$.

(f) $\lim_{z\to 9}\dfrac{3-\sqrt{z}}{9-z}=\lim_{z\to 9}\left(\dfrac{3-\sqrt{z}}{9-z}\times\dfrac{3+\sqrt{z}}{3+\sqrt{z}}\right)=\lim_{z\to 9}\dfrac{9-z}{(9-z)(3+\sqrt{z})}=\lim_{z\to 9}\dfrac{1}{3+\sqrt{z}}=\dfrac{1}{6}$. Notice that we could not directly set $z=9$ as this leads to the nonsense $\left[\dfrac{0}{0}\right]$, but after some algebra it became posible. Setting $z=9$ in the last step is valid since the function $\dfrac{1}{3+\sqrt{z}}$ is continuous at $z=9$.

2. Find an equation of the tangent line of $y=f(x)$ at $x=a$ for the following:

(a) $y=x^2+x,\ a=2$ (b) $y=\dfrac{x+1}{x-1},\ a=3$ (c) $y=\sqrt{x+1},\ a=3$

SOLUTIONS:

The equation of the tangent line to the graph of $y=f(x)$ at $x=a$ is always given by $y=f(a)+f'(a)(x-a)$. We use the laws of differentiation to compute the derivatives.

(a) If $f(x) = x^2 + x$ then $f'(x) = 2x + 1$, and so the equation of the tangent line is $y = 6 + 5(x - 2)$.

(b) If $f(x) = \dfrac{x+1}{x-1}$ then $f'(x) = \dfrac{-2}{(x-1)^2}$, and so the equation of the tangent line is $y = 2 - \dfrac{1}{2}(x - 3)$.

(c) If $f(x) = \sqrt{x+1}$ then $f'(x) = \dfrac{1}{2\sqrt{x+1}}$, and so the equation of the tangent line is $y = 2 + \dfrac{1}{4}(x - 3)$.

3. Evaluate the following limits:

(a) $\displaystyle\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{\sqrt{1+h}} - 1\right)$ (b) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

SOLUTIONS:

(a)

$$\begin{aligned}
\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{\sqrt{1+h}} - 1\right) &= \lim_{h\to 0} \frac{1}{h}\left(\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\right) = \lim_{h\to 0}\left(\frac{1}{h} \times \frac{1-\sqrt{1+h}}{\sqrt{1+h}} \times \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}\right) \\
&= \lim_{h\to 0}\left(\frac{1}{h} \times \frac{1-(1+h)}{\sqrt{1+h}(1+\sqrt{1+h})}\right) = \lim_{h\to 0} \frac{-1}{\sqrt{1+h}(1+\sqrt{1+h})} \\
&= -\frac{1}{2}
\end{aligned}$$

(b)

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} &= \lim_{x\to 0}\left(\frac{\sqrt{x+4}-2}{x} \times \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right) = \lim_{x\to 0} \frac{(x+4)-4}{x(\sqrt{x+4}+2)} \\
&= \lim_{x\to 0} \frac{x}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{4}
\end{aligned}$$

4. A certain function $y = f(x)$ satisfies $f(1) = -1, f'(1) = 2$.

(a) Determine an equation for the tangent line at $x = 1$.
(b) Find the $x$ and $y$ intercepts of the tangent line.
(c) Graph the tangent line.

SOLUTIONS:

(a) The equation is $y = f(1) + f'(1)(x - 1)$, that is $y = -1 + 2(x - 1)$.
(b) The $x$ and $y$ intercepts are $x = 3/2$ and $y = -3$ respectively.

(c)

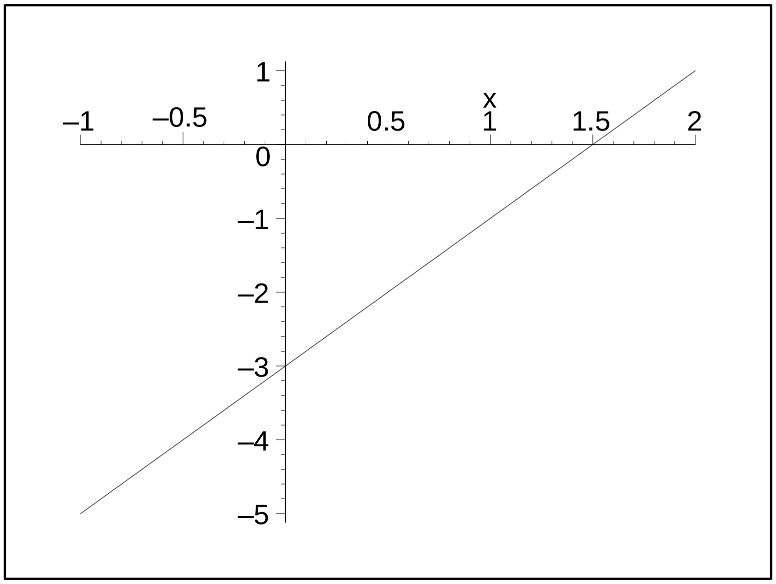


Figure 1: The tangent line of $y = f(x)$ at $x = 1$