Name: Student number:

Math 253 Midterm Exam

1. (a) Consider the line with parametric equations: $x = 2 + 2t, \quad y = 1 + 5t, \quad z = 4t$. Find the point in which this line meets the plane: $x - y + z = 3$. [6]

(b) Find the equation of the line perpendicular to the above line, parallel to the above plane, and passing through the point $(x, y, z) = (2, 3, -2)$. [7]

(c) Find the distance between the above plane and the origin. [7]

2. Let $h(x, y) = 4x^2 + 4y^2$.

(a) Sketch the surface $z = h(x, y)$. [5]

(b) Write the equation for this surface in cylindrical coordinates $z = z(r, \theta)$. [5]

(c) Write the equation for this surface in spherical coordinates $\rho = \rho(\theta, \phi)$ [5]

(d) Find the equation of the plane tangent to the surface $z = h(x, y)$ at the point $(x, y, z) = (1, 1, 8)$. [5]

3. Find and classify the critical points of $f(x, y) = x^3 - y^3 - 2xy + 6$. [20]

4. (a) Calculate the gradient of the function $g(x, y) = xe^{xy}$. [6]

(b) At the point $(1, 0)$, in what direction will g decrease most rapidly? [7]

(c) What is the directional derivative in that direction? [7]

5. Find the maximum and minimum values of the function $F(x, y) = xy$ on the ellipse defined by $x^2 + 4y^2 = 4$, and the points at which the max and min occur. [20]