HOMEWORK ASSIGNMENT #7

1. The iterated integral $I = \int_{x=0}^{x=1} \left(\int_{y=0}^{y=\sqrt{x}} \sin\left(\frac{\pi(y^3 - 3y)}{2} \right) dy \right) dx$ is equal to the double integral $\int\int_R \sin\left(\frac{\pi(y^3 - 3y)}{2} \right) dA$ for a region R in the x, y plane.

   (a) Sketch R.

   (b) Write the integral with the order of integration reversed.

   (c) Compute I.

2. Let D be the region bounded by $y = x$ and $y = 6 - x^2$.

   (a) Sketch D.

   (b) Find $\int\int_D x^2 dA$.

3. Let D be the region, described in polar coordinates by, $0 \leq \theta \leq \pi, 0 \leq r \leq 1 + \cos\theta$.

   (a) Sketch D.

   (b) Compute the area of D.

   (c) Find the average value of distances of points in D from the origin.

4. Determine the following integrals:

   (a) $\int\int_D (|x| + |y|) dA$, where D is the region $x^2 + y^2 \leq a^2$ and a is a positive constant.

   (b) $\int\int_T \sqrt{a^2 - x^2} dA$, where T is the triangle with vertices $(0, 0), (a, 0), (a, a)$.

   (c) $\int\int_D \frac{1}{x^2 + y^2} dA$, where D is the region in the first quadrant bounded by

   $$y = 0, y = x, x^2 + y^2 = 1/4, x^2 + y^2 = 1.$$

   (d) $\int\int_R \left(\sin xy + x^2 - y^2 + 3 \right) dxdy$, where R is the region inside the circle $x^2 + y^2 = a^2$ and outside the circle $x^2 + y^2 = b^2$, and a, b are constants satisfying $0 < b < a$.

5. Find the volume above the x, y plane, below the surface $z = e^{-(x^2+y^2)}$ and inside the cylinder $x^2 + y^2 = 4$.

6. Find the volume above the x, y plane and below the surface $z = e^{-(x^2+y^2)}$.

7. Compute the double integral $\int\int_D (x + y)\ dA$, where D is the domain that lies to the right of the y-axis and between the circles $x^2 + y^2 = 1,\ x^2 + y^2 = 4$.

8. Find the area that is common to the polar curves $r = \cos\theta,\ r = \sin\theta$.

9. Find the area that is inside the polar curve $r = 4\sin\theta$ and outside the circle $r = 2$.

10. Find the volume that is above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2+y^2+z^2 = 1$.

11. A cylindrical hole of radius a is drilled through a sphere of radius b ($a < b$). Find the volume of the solid that remains.