HOMEWORK ASSIGNMENT 3, Math 253

1. Calculate the following limits, or discuss why they do not exist:
   (a) $\lim_{(x,y)\to 0} \dfrac{y}{x^2+y^2}$
   (b) $\lim_{(x,y)\to 0} \dfrac{y^3}{x^2+y^2}$ [hint: $|y^2| \le |x^2+y^2|$]

2. For each of the following functions, give its domain and calculate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$:
   (a) $f(x,y) = e^{3x}\cos(3y)$
   (b) $f(x,y) = \ln(1+xy^2)$
   (c) $f(x,y) = \dfrac{y}{x^2+y^2}$
   (d) $f(x,y) = x^y$
   (e) $f(x,y) = \cosh(x)\cos(y)$
   (f) $f(x,y) = x^3 \arcsin y^2$

3. Which of the above functions satisfies the Laplace equation: $f_{xx} + f_{yy} = 0$?

4. Give an equation for the tangent plane to the graph of $f(x,y) = \dfrac{y}{x^2+y^3}$ at the point $(0,1,1)$. What is the normal vector at that point?

5. Find the coordinates of all points at which the surface with the following equation has a horizontal tangent plane: $z = x^4 - 4xy^3 + 6y^2 - 2$.

6. The equation $x^3y^4 + xz^2 - yz^3 = 1$ defines a surface which passes through the point $(1,1,1)$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at that point.

7. The radius and height of a right circular-conical tank are measured and found to be 25 ft and 21 ft respectively. Each measurement is accurate to within 0.5 in. By about how much can the calculated volume of the tank be in error?

8. Write an appropriate version of the chain rule for $\dfrac{\partial z}{\partial u}$, if $z = g(x,y)$, $y = f(x)$ and $x = h(u,v)$.

9. Use two different methods to calculate $\dfrac{\partial z}{\partial x}$ given that
$$z = \arctan(u/v), \quad u = 2x + y, \quad v = 3x - y.$$

10. Calculate $\dfrac{\partial}{\partial x} f(y^2, x^2)$, assuming f has continuous partial derivatives.