HOMEWORK ASSIGNMENT #5, Math 253

1. For what values of the constant k does the function $f(x, y) = kx^3 + x^2 + 2y^2 - 4x - 4y$ have

   (a) no critical points;

   (b) exactly one critical point;

   (c) exactly two critical points?

   Hint: Consider $k = 0$ and $k \neq 0$ separately.

2. Find and classify all critical points of the following functions.

   (a) $f(x, y) = x^3 - y^3 - 2xy + 6$

   (b) $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$

   (c) $f(x, y) = \frac{1}{x^2+y^2-1}$

   (d) $f(x, y) = y \sin x$

3. Suppose $f(x, y)$ satisfies the Laplace's equation $f_{xx}(x, y) + f_{yy}(x, y) = 0$ for all x and y in $\mathbb{R}^2$. If $f_{xx}(x, y) \neq 0$ for all x and y, explain why $f(x, y)$ must not have any local minimum or maximum.

4. Find all absolute maxima and minima of the following functions on the given domains.

   (a) $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate with vertices $(0, 0)$, $(2, 0)$, and $(2, 2)$

   (b) $f(x, y) = x^2 + xy + 3x + 2y + 2$ on the domain $D = \{(x, y)|x^2 \leq y \leq 4\}$

   (c) $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the domain $D = \{(x, y)|x^2 + y^2 \leq 16\}$

5. Use Lagrange multipliers to find the maximum and minimum values of the following functions subject to the given constraint(s).

   (a) $f(x, y) = xy^2$ subject to $x^2 + 2y^2 = 1$

   (b) $f(x, y, z) = xy + z^2$ subject to $y - x = 0$ and $x^2 + y^2 + z^2 = 4$