Math 253 Homework assignment 6

1. Consider the integral $\int_R xy^2 dA$, where A is the rectangle $[0,1] \times [0,1]$.

   (a) Calculate the Riemann sum corresponding to this integral, with the subdivision corresponding to $\Delta x = \Delta y = 0.2$ and using the centre of each small rectangle as the sample point (x^*_{ij}, y^*_{ij}).

   (b) Using an iterated integral, calculate the value exactly.

2. Find the volume of the solid bounded by the planes $x = 1$, $x = 2$, $y = 0$, $y = \pi/2$, $z = 0$ and the surface $z = x \cos y$.

3. Calculate $\iint_R \sqrt{x+y} dA$, where $R = [0,1] \times [0,3]$.

4. Find $\iint_R (x^2 + y^2) dA$, where R is the rectangle $0 \leq x \leq a, \quad 0 \leq y \leq b$.

5. Calculate the iterated integral $\int_0^{\pi} \int_{-x}^{x} \cos y dy dx$.

6. Find the volume under the surface $z = \dfrac{1}{x+y}$ and above the region in the xy-plane bounded by $x = 1$, $x = 2, y = 0$ and $y = x$.

7. Using a double integral, calculate the volume of the tetrahedron in the first quadrant bounded by the coordinate planes and the plane which intersects the x- y- and z-axes at a, b and c, respectively, where a, b, c are positive numbers.

8. Calculate the integral $I = \int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dy dx$.