HOMEWORK ASSIGNMENT #9, Math 253

1. For each of the following regions E, express the triple integral $\iiint_E f(x,y,z)\,dV$ as an iterated integral in cartesian coordinates.

   (a) E is the box $[0,2] \times [-1,-1] \times [3,5]$;

   (b) E is the pyramid with vertices $(0,0,0)$, $(1,1,1)$, $(1,1,-1)$, $(-1,1,1)$, and $(-1,1,-1)$;

   (c) E is the region in the first octant bounded by the cylinder $x^2+z^2=1$ and the plane $y=z$.

   (d) E is the region inside the sphere $x^2+y^2+z^2=2$ and above the elliptic paraboloid $z=x^2+y^2$.

2. Consider the integral

$$\iiint_E f(x,y,z)\,dV = \int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{y} f(x,y,z)\,dz\,dy\,dx$$

   (a) Sketch the region E.

   (b) Write the other five iterated integrals which represent $\iiint_E f(x,y,z)\,dV$.

   (c) Find the volume of E.

   (d) Find the centre of mass of E when the density of E is constant.

3. Let E be the solid bounded by $z=\sqrt{x^2+y^2}$ and $z=\sqrt{1-x^2-y^2}$,

   (a) Use cylindrical coordinates to find the volume of E.

   (b) Use spherical coordinates to find the volume of E.

4. Find the volume of the solid above the xy-plane, under the surface $z=1-x^2-y^2$, and within the wedge $x \le y \le \sqrt{3}x$.

5. Find the volume remaining in a sphere of radius a after a hole of radius b is drilled through the centre. Assume $0<b<a$.

6. Find the mass of the solid between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ above the xy-plane when the density is $\rho(x,y,z)=z$.