SOLUTIONS TO QUIZ #2, Math 253

1. (a) Find the point of intersection the two lines $L_1: \ x = 1 + 2t, \ y = 1 - t, \ z = 1 + 2t$ and $L_2: \quad x = 2 + s, \ y = 2 + s, \ z = 3 + 2s$

   (b) Find an equation of the plane containing both lines.

   Solution:

   (a) We find the point of intersection by finding s, t such that

   $$1 + 2t = 2 + s, 1 - t = 2 + s, 1 + 2t = 3 + 2s.$$

   By inspection there is a solution, namely $t = 0, s = -1$. Thus the point of intersection is $(x, y, z) = (1, 1, 1)$.

   (b) The direction vectors for the lines are $\vec{v_1} = 2\vec{i} - \vec{j} + 2\vec{k}, \ \vec{v_2} = \vec{i} + \vec{j} + 2\vec{k}$, and therefore a normal for the plane containing both lines is

   $$\vec{n} = (2\vec{i} - \vec{j} + 2\vec{k}) \times (\vec{i} + \vec{j} + 2\vec{k}) = -4\vec{i} - 2\vec{j} + 3\vec{k}.$$

   The equation for the plane is thus $-4(x - 1) - 2(y - 1) + 3(z - 1) = 0$.

2. Let V be the region in 3-space consisting of all points (x, y, z) satisfying the inequalities $\sqrt{x^2 + y^2} \leq z \leq \sqrt{1 - x^2 - y^2}$. Sketch the region V and describe it in spherical coordinates.

   Solution: The sketch is on the next page. In spherical coordinates the region is given by $0 \leq \rho \leq 1, \ 0 \leq \phi \leq \pi/4, \ 0 \leq \theta \leq 2\pi$.

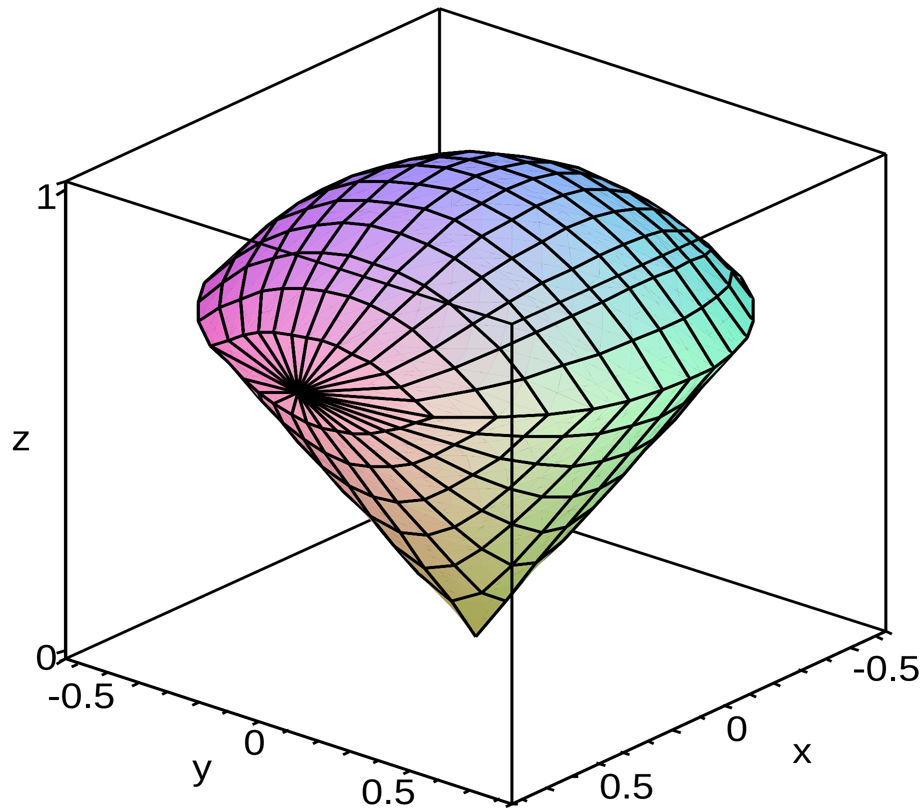

Figure 1: The region $\sqrt{x^2+y^2} \le z \le \sqrt{1-x^2-y^2}$

The side $z = \sqrt{x^2+y^2}$ is a cone with $\phi = \pi/4$ and the top is part of the sphere $\rho = 1$.