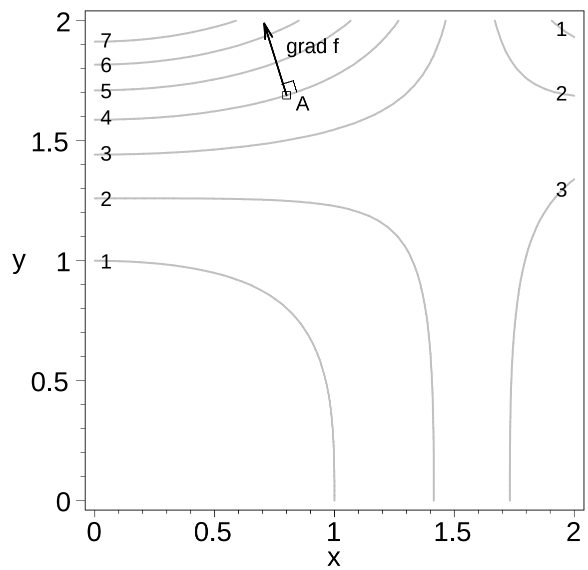
SOLUTIONS TO QUIZ #3, Math 253

1. Given the contour plot of $f(x, y)$,



   (a) plot the direction of ∇f at point A <u>on the diagram</u>;

   (b) state whether the following quantities are positive or negative at point A:

      i. $\frac{\partial f}{\partial x}$, $\boxed{-}$

      ii. $\frac{\partial f}{\partial y}$, $\boxed{+}$

      iii. derivative of f in the direction of the vector $\langle -1, -2 \rangle$, $\boxed{-}$

      iv. $\frac{dy}{dx}$ along the level curve $f(x, y) = 4$. $\boxed{+}$

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2. Find the equations of the tangent plane and normal line to the surface $z = \ln(xy - y^4)$ at the point $(x, y, z) = (2, 1, 0)$.

   Solution:

   Set $f(x, y, z) = \ln(xy - y^4) - z = 0$. Choose $\vec{n} = \nabla f(2, 1, 0)$ to be the normal vector of the tangent plane, and the parallel vector of the normal line.

$$\nabla f = \langle f_x, f_y, f_z \rangle = \left\langle \frac{y}{xy - y^4}, \frac{x - 4y^3}{xy - y^4}, -1 \right\rangle$$

   At $(2, 1, 0)$, $\nabla f(2, 1, 0) = \langle 1, -2, -1 \rangle$.

   Hence the tangent plane is $\boxed{(x-2) - 2(y-1) - z = 0}$,

   and the normal line is $\boxed{\vec{r} = \langle 2, 1, 0 \rangle + t\langle 1, -2, -1 \rangle}$.

3. Suppose $\nabla f = \langle 1, -2 \rangle$ at point P.

(a) Find the derivative of f at point P in the direction of the vector $\langle 3, 4 \rangle$.

Solution:

Set $\vec{u} = \langle 3, 4 \rangle$. The unit vector $\hat{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{\langle 3,4 \rangle}{\sqrt{3^2+4^2}} = \langle \frac{3}{5}, \frac{4}{5} \rangle$. Hence,

$$D_{\hat{u}} f = \nabla f \cdot \hat{u} = \langle 1, -2 \rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle = \boxed{-1}$$

(b) Find a unit vector $\vec{v}$ so that the derivative of f at point P in the direction $\vec{v}$ is 0.

Solution:

Let $\vec{v} = \langle a, b \rangle$ be a unit vector. Then $D_{\vec{v}} f = \nabla f \cdot \vec{v} = \langle 1, -2 \rangle \cdot \langle a, b \rangle = a - 2b = 0$, which gives $a = 2b$ and $\vec{v} = \langle 2b, b \rangle$. Since $\vec{v}$ is a unit vector, $|\vec{v}| = \sqrt{(2b)^2 + b^2} = \sqrt{5b^2} = 1$, we have $b = \pm\frac{1}{\sqrt{5}}$. So $\boxed{\vec{v} = \langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle}$ or $\boxed{\langle -\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \rangle}$.