SOLUTIONS TO QUIZ #4, Math 253

1. Determine the volume of the solid that lies above the rectangle $0 \le x \le 1, 0 \le y \le 2$ in the x, y plane and below the surface $z = 5 - x^2 - y^2$.

   Solution: Notice that $5 - x^2 - y^2 \ge 0$ on the rectangle $0 \le x \le 1,\ 0 \le y \le 2$. The volume is

   $$V = \int_{x=0}^{x=1} \int_{y=0}^{y=2} (5 - x^2 - y^2)\, dydx = \int_{x=0}^{x=1} \left(10 - 2x^2 - 8/3\right) dx = 10 - 2/3 - 8/3 = 20/3$$

2. Evaluate the double integral $\int\int_D 6xy\ dA$, where D is the triangle with vertices $(0,0)$, $(1,1)$ and $(0,2)$.

   Solution:

   $$\begin{aligned} \int\int_D 6xy\ dA &= \int_{x=0}^{x=1} \int_{y=x}^{y=-x+2} 6xydydx = \int_{x=0}^{x=1} 3xy^2 \Big|_{y=x}^{y=-x+2} dx \\ &= \int_{x=0}^{x=1} 3x(-4x+4)dx = -4 + 6 = 2 \end{aligned}$$

3. Evaluate the iterated integral $\displaystyle\int_{y=0}^{y=1} \int_{x=0}^{x=y} 2x\sqrt{1+y^3}\ dxdy$

   Solution:

   $$\int_{y=0}^{y=1} \int_{x=0}^{x=y} 2x\sqrt{1+y^3}\ dxdy = \int_{y=0}^{y=1} y^2\sqrt{1+y^3} = \frac{2}{9}\left(1+y^3\right)^{3/2} \Big|_{y=0}^{y=1} = \frac{2}{9}(2\sqrt{2} - 1)$$