Math 257-316 Assignment 1

1. Copy down the secret code in the email which you received from me on Friday at the email account you registered with UBC. (The purpose of this problem is to make sure you will receive emails from me in the future.)

   [**Solutions of linear equations**]

2. Suppose that u_1 and u_2 are solutions of the heat equation
$$\frac{\partial}{\partial t}u = \frac{\partial^2}{\partial x^2}u - 2u \tag{1}$$
for $u = u(x,t)$. Show that $v = c_1 u_1 + c_2 u_2$ is also a solution, where c_1 and c_2 are constants.

3. Derive the general solution of the equation $2\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = 0$. Find the solution if it is equal to e^{-x^2} along the x-axis at time $t = 0$.

4. Let F and G be arbitrary twice differentiable functions of one variable. Show that $u(x,t) = F(x+ct) + G(x-ct)$ is a solution to the wave equation
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{2}$$

5. Verify that the function
$$u(x_1, x_2, x_3) = \frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$
is a solution to the three dimensional Laplace equation. (Note that it is everywhere smooth except at the origin. It is called the *fundamental solution of the Laplace equation in* $\mathbb{R}^3$.)

   [**Taylor's theorem and power series**]

6. Let $f(x) = \frac{1}{1+4x^2}$. Expand it as a Taylor series centered at $x = 0$, and find its radius of convergence.

7. Let $f(x) = \frac{1+x}{1-x+2x^2}$. Find the first four nonzero terms of its Taylor series centered at $x = 0$. Also find the first three nonzero terms of the power series of $f'(x)$ and $F(x) = \int_0^x f(t)\,dt$.

- This assignment is due at the beginning of class of Wednesday, September 14.